

## ONE MATHEMATIC EQUATION FOR ASSESSMENT OF MAXIMUM LIGHTNING OVERVOLTAGE ALONG A CABLE

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**Abstract-** Lightning is a factor of producing overvoltages in transmission lines especially cable lines. In this paper, we have used forward and backward voltage waves algorithm for assessment of maximum lightning overvoltage along a cable. Maximum overvoltage has been calculated as function of surge impedance of overhead line connected to cable, cable surge impedance and its length. Finally, a mathematic equation for assessment of maximum overvoltage has been evaluated.

**Keywords:** Maximum Overvoltage, Insulated Cable, Transient.

### I. INTRODUCTION

The lightning strike to transmission lines is a phenomenon that power systems experiences constantly. Lightning as the most significant external source producing transient overvoltage in power systems is of great significance. Hence there has been a great deal of attempt to calculate the amount and quality of distribution of lightning overvoltage on transmission line for the purpose of insulation coordination and protecting these lines [1, 2 and 3].

Among the main methods used in this case is the solving the transmission line equations considering the effect of lightning that is done approximately and uses numerical solution [4, 5, 6 and 7]. ANN is another method for obtaining the maximum lightning overvoltage with considering effects of different parameters [8]. Also, maximum overvoltage can be found by calculating the overvoltage at selected positions along the cable. This can be done by subdividing the cable into a number of small sections [9].

We have used forward and backward voltage waves at the end of cable for calculation of maximum overvoltage [10]. Then final equation has been obtained by statistic data and fitting an equation for this data. In final part of paper, assessment of final equation and its error in comparison to forward and backward waves algorithm is analysed.

### II. COMPUTATIONAL PROCEDURE

Maximum lightning overvoltage was calculated based on the method presented in [10]. The overvoltage at the

any point along the cable is sum of forward and backward voltage waves:

$$V(x,t) = V_+(x,t) + V_-(x,t) \quad (1)$$

Arrester current at the remote end of cable is calculated by equation (2):

$$i_{arr}(t) = (2V_+(l,t)/Z_C) - (V(l,t)/Z_C) - (V(l,t)/Z_L) \quad (2)$$

In this equation  $Z_C$ ,  $Z_L$ ,  $V_+(l,t)$ ,  $V(l,t)$  and  $i_{arr}(t)$  are surge impedances of cable and overhead line, forward voltage wave, overvoltage and arrester current at the remote end of cable.

Arrester current and overvoltage at the cable end can be calculated by ATP/EMTP program. Then, equation (2) gives the forward wave. Finally, backward wave is calculated by equation (1). Considering [10] the maximum overvoltage along the cable is sum of the peak values for forward and backward voltage waves.

### III. SYSTEM CONFIGURATION

System is a cable inserted in an overhead line. This system has been shown in Figure 1. Figure 2 shows configuration of system in details. The arrester characteristic is shown in Table 1 and the leads are represented by inductances based on  $1\mu\text{H}/\text{m}$  and  $7\text{m}$  length.

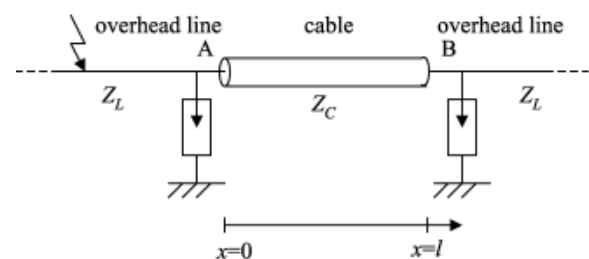


Figure 1. System configuration

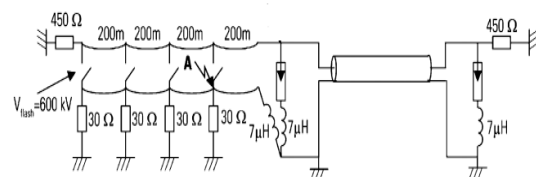


Figure 2. Configuration of system in details

Table 1. Surge arrester current-voltage characteristic

Current (A)	Voltage (kV)
0.0001	172.02
0.00099996	194.52
0.0099981	206.65
0.10002	218.82
0.99991	234.59
250	308.67
1000	331.03
4999.8	369.53
10000	395.00
20000	435.99

We use four spans for overhead line and assume that the line is lossless. Each of them are represented by a two conductor distributed parameter model. A single core and jmarti model is used for cable. The data of cable for different impedances are in Table 2.

A flashover is assumed to take place at each tower except for the tower closest to the cable entrance, when the magnitude of the voltage between phase conductor and tower exceeds 600kV. The voltage drop along the corresponding towers is not taken into account, but a constant footing resistance of 30Ω is included as shown in the Figure 3.

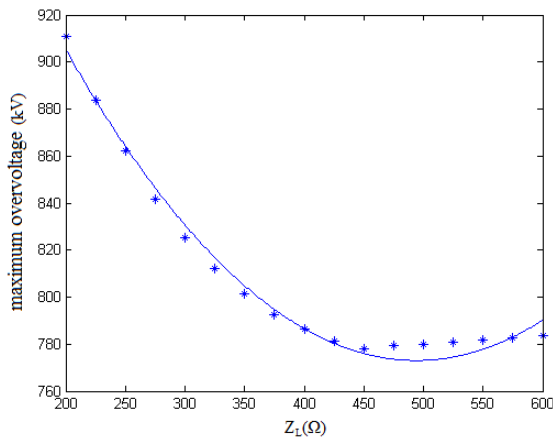


Figure 3. Maximum overvoltage as function of characteristic impedance of overhead line

IV. ANALYSIS

In this section, we have used the method in section II and calculated maximum overvoltage for different parameters. Then we have fitted an equation for values of maximum overvoltage along the cable for different values of overhead line surge impedance, cable surge impedance and its length by MATLAB. Finally, an equation as function of surge impedance of cable and overhead line connected to the cable and its length has been evaluated.

A. Maximum Overvoltage as Function of Characteristic Impedance of the Overhead Line Connected to Cable

For different amounts of the characteristic impedance of overhead line, the maximum overvoltage along the cable has been calculated and shown in Figure 3. As this figure shows, the data has been fitted by a quadratic function. This function is:

$$V_{max} = 0.0015306Z_L^2 - 1.5125Z_L + 1146.7 \tag{3}$$

In this equation  $V_{max}$  and  $Z_L$  are maximum overvoltage along cable and characteristic impedance of overhead line. Increasing the characteristic impedance of overhead line causes reduction of ratio of cable impedance to it. Therefore peak value of the impinging voltage reduces.

B. Maximum Overvoltage as Function of Characteristic Impedance of the Cable

We have used jmarti model for cable, as mentioned in Appendix. Therefore, impedance of cable changes by changing parameters of cable such as radius of core and thickness of the insulator material outside the core. So, an Equation as function of cable impedance includes these parameters. Table 2 shows impedance of cable for different parameters. For cable impedances shown in Table 2, maximum overvoltage has been calculated and is shown in Figure 4.

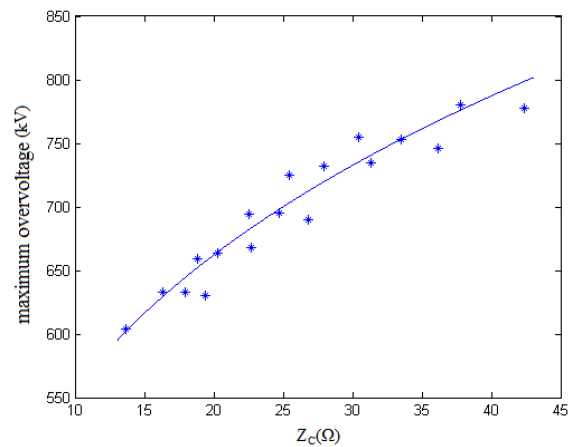


Figure 4. Maximum overvoltage as function of characteristic impedance of cable

By fitting this data by MATLAB, The maximum overvoltage as function of cable impedance is:

$$V_{max} = 313\sqrt{Z_C} \tag{4}$$

In equation (4),  $V_{max}$  is the maximum overvoltage and  $Z_C$  is the characteristic impedance of cable. Increasing cable impedance causes the increase of its ratio to overhead line impedance and because of this; the peak value of the impinging voltage reduces. So, maximum overvoltage increases.

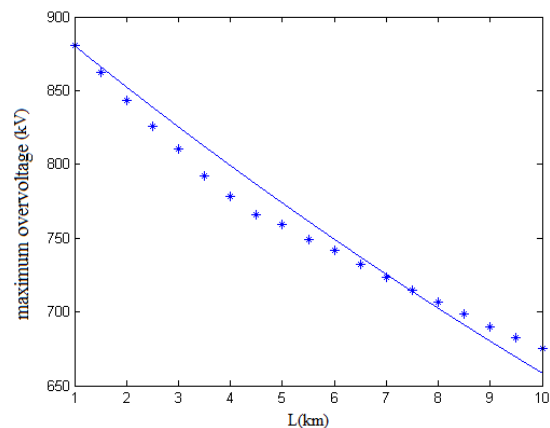


Figure 5. Maximum overvoltage as function of cable length

Table 2. Impedance of cable for different parameters

radius of core (mm)	thickness of the insulation (mm)	thickness of the sheath (mm)	Relative permittivity of the insulation	Z <sub>c</sub> (Ω)
13	25	2	2.3	42.41
16.5	26.4	2	2.3	37.78
12	18	2	2.3	36.2
15	20	2	2.3	33.51
13	25	2	4.2	31.3
19	22	4	2.3	30.41
16.5	26.4	2	4.2	27.96
12	18	2	4.2	26.81
21	19	2	2.3	25.48
15	20	2	4.2	24.75
13	25	2	8	22.74
19	22	4	4.2	22.5
16.5	26.4	2	8	20.26
12	18	2	8	19.43
21	19	2	4.2	18.85
15	20	2	8	17.96
19	22	4	8	16.31
21	19	2	8	13.66

**C. Maximum Overvoltage as Function of Length of Cable**

The maximum overvoltages for cable lengths from 1km to 10km have been shown in Figure 5. The maximum overvoltage decreases with increasing the cable length due to increase of attenuation along the cable. By fitting of data in Figure 5 the maximum overvoltage as function of cable length equals:

$$V_{max} = 909.5729e^{\frac{L}{31}} \tag{5}$$

In the equation (5), V<sub>max</sub> and L are maximum overvoltage along the cable and length of the cable.

**D. The Final Equation and Assessment**

The final equation calculated by multiply equations in parts A, B and C together and unknown and invariable coefficient of k:

$$V_{max} = k(0.0015306Z_L^2 - 1.5125Z_L + 1146.7)e^{\frac{L}{31}}\sqrt{Z_C} \tag{6}$$

200 ≤ Z<sub>L</sub> ≤ 600  
 13.66 ≤ Z<sub>C</sub> ≤ 42.41  
 1 ≤ L ≤ 10

In this equation k is a coefficient the amount of which for different parameters has been shown in table 3. The average amount of Table 3 is 0.4552 and we have selected it for k. so, the final equation has been corrected and shown in equation (7):

$$V_{max} = 0.4552(0.0015306Z_L^2 - 1.5125Z_L + 1146.7)e^{\frac{L}{31}}\sqrt{Z_C} \tag{7}$$

200 ≤ Z<sub>L</sub> ≤ 600  
 13.66 ≤ Z<sub>C</sub> ≤ 42.41  
 1 ≤ L ≤ 10

Testing of this equation for different parameters of system has been done and shown in Table 4. Error in this table has been calculated with equation (8):

$$\%error = \frac{|\text{forward and backward method-equation}|}{\text{forward and backward method}} \tag{8}$$

Table 4 shows that the error is less than 6% and considering ease of equation (7) for obtaining maximum overvoltage along the cable, this equation is much better than forward and backward waves method for the system which we have used in this paper.

Table 3. The amount of k for different parameters

Z <sub>c</sub> (Ω)	Z <sub>L</sub> (Ω)	L (km)	maximum overvoltage (kV)	k
42.41	200	4	910.9	0.4485
20.26	450	4	663.7	0.4586
42.41	450	6	741.6	0.4544
37.78	500	5	761.1	0.4666
26.81	450	4	690.2	0.4447
22.74	400	6	632.1	0.4466
42.41	450	1	880.7	0.4593
42.41	450	10	674.9	0.4705
42.41	600	4	783.6	0.4421
13.66	450	4	604.3	0.4608

**V. CONCLUSIONS**

In this paper, maximum lightning overvoltage as a function of significant three parameters (characteristic impedance of overhead line and cable, and length of cable) was obtained using statistic analysis of maximum overvoltage amounts and fitting of them with equation by MATLAB.

The k coefficient in the final equation is between 0.4421 and 0.4705. We used average of these amounts which was 0.4552. The comparing the final equation with forward and backward waves algorithm showed that error of it is less than 6%. Considering simplicity and high accuracy of this equation, it is an excellent method for assessment maximum lightning overvoltage along a cable.

Table 4. Assessment of the equation for different parameters

$Z_L$ ( $\Omega$ )	$Z_C$ ( $\Omega$ )	$L$ (km)	Maximum overvoltage (kV)		Error (%)
			Forward and backward waves method	Equation (5)	
300	42.41	4	825.4	848.16	2.76
450	30.41	4	754.5	729.11	3.37
450	42.41	2	843.5	845.13	0.19
450	42.41	10	674.9	652.9	3.26
600	42.41	4	783.6	806.82	2.96
500	37.78	5	761.1	742.52	2.44
400	22.74	6	632.1	644.31	1.93
550	31.3	4	734.1	736.1	0.27
200	22.5	3	772	814.83	5.55
350	16.31	7	599.6	587.42	2.03
275	13.66	2	660.7	694.51	5.12
475	36.2	5	722.6	735.11	1.73
575	26.81	9	606.5	606.72	0.04
475	24.75	6	648.4	647.23	0.18
525	36.2	6	699.6	712.61	1.86
325	42.41	4	812	833.97	2.71
450	33.51	4	752.8	747.02	0.77
450	42.41	8.5	698.4	685.27	1.88
450	42.41	1.5	862.5	858.87	0.42
225	42.41	4	883.6	902.45	2.13

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**BIOGRAPHIES**



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