INVESTIGATION OF THE INFLUENCE OF PRELIMINARY BUCKLING OF CYLINDRICAL SHELL REINFORCED BY A CROSS SYSTEM OF RIBS AND FILLED WITH MEDIUM ON CRITICAL STRESSES OF GENERAL STABILITY LOSS

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Abstract - Influence of preliminary buckling of a shell reinforced by a regular cross system of ribs and filled with medium on critical load parameters of general stability loss is investigated. The investigation is based on the problem statement used the mixed energy method and nonlinear equation of combined deformations.

Keywords: Damaged Shell, Ribbing, Vibrations, Viscous-Elastic Medium, Elastic Matrix.

I. INTRODUCTION

Cylindrical shells reinforced by a regular cross system of ribs are important structural elements of rockets, submarines, motor vehicles and etc. Investigation of the behavior of such structures with regard to external factors is of special importance in the field of contact problems in the theory of ribbed shells. In papers [1-3], the stability under longitudinal compression was considered without taking into account the preliminary buckling of ribbed cylindrical shells filled with medium.

II. PROBLEM STATEMENT

Total energy of the system is written in the form [4]:

\[ \Pi = \mathcal{E} + A \]

where

\[ \mathcal{E} = \frac{Eh^3}{24(1-v^2)} \left[ \left( \frac{\partial^2 w}{\partial \xi^2} \right)^2 + \left( \frac{\partial^2 w}{\partial \eta^2} \right)^2 \right] - 2(1-v) \left[ \frac{\partial^2 w}{\partial \xi \partial \eta} \right] d\xi d\eta \]

\[ + \frac{h}{2E} \left( \left( \frac{\partial^2 w}{\partial \xi^2} \right)^2 - 2(1-v) \left( \frac{\partial^2 w}{\partial \eta^2} \right)^2 \right) d\xi d\eta \]

\[ + \frac{1}{2} \sum_{i=1}^{N} E_i F_i I_{ki} \left( \frac{\partial^2 w}{\partial \xi^2} \right) + G_i I_{ki} \left( \frac{\partial^2 w}{\partial \eta^2} \right) d\xi + \sigma_i \frac{h^3}{2E} \left( \frac{\partial^2 w}{\partial \xi^2} \right)^2 d\xi \]

\[ + \frac{1}{2} \sum_{i=1}^{N} E_i F_i I_{ki} \left( \frac{\partial^2 w}{\partial \eta^2} \right) d\xi + G_i I_{ki} \left( \frac{\partial^2 w}{\partial \eta^2} \right) d\eta \]

and \( \xi = \frac{x}{r}, \ \theta = \frac{y}{r} \); \( E_r, G_r, E_v, G_v \) are elastic modulus and shear modulus of the material of longitudinal ribs; \( k, k_i, k_{ii} \) are numbers of longitudinal and lateral ribs, respectively; \( \sigma \) is axial compression stresses; \( u, v, w \) are components of the displacement vector of the shell; \( h \) and \( R \) are the thickness and radius of the shell, respectively; \( E_v \) and Young’s modulus and Poisson ratio of the shell material; \( \xi_0 = \frac{L_i}{L} \), \( L_i \) is the shell length, \( r \) is the radius of the shell.

The influence of medium on the shell is determined as external surface loads applied to the shell and is calculated as a work performed by these loads when the system changes from strained state to the initial unstrained state. It is represented in the form:

\[ A = -R^2 \left[ \sum_{i=1}^{N} q_i \left( \frac{\partial w}{\partial \xi} \right) + \sum_{i=1}^{N} q_{i\eta} \left( \frac{\partial w}{\partial \eta} \right) \right] d\xi + \sum_{i=1}^{N} \left( \frac{\partial w}{\partial \xi} \right) \left( \frac{\partial w}{\partial \eta} \right) d\xi + \sum_{i=1}^{N} \left( \frac{\partial w}{\partial \xi} \right) \left( \frac{\partial w}{\partial \eta} \right) d\eta \]

The Pasternak model [5] is used to determine \( q_i \). The essence of this model is that the influence of medium on the shell on the contact surface is determined by the relation

\[ q_i = (\tilde{q} + \tilde{q}_0 \nabla^2) w = Kw \]

where \( \nabla^2 \) is Laplace’s two-dimensional operator on the contact surface. Strain continuity equation for \( w_0 = 0 \) is written in the form of relation (4).

\[ \Delta \Delta \varphi = E \left[ \left( \frac{\partial^2 w}{\partial \xi^2} \right)^2 + \left( \frac{\partial^2 w}{\partial \eta^2} \right)^2 \right] \]
III. METHOD OF SOLUTION

It is accepted that the end faces of the shell are simply supported. It is assumed that before buckling of the shell, the given axial compression stresses are uniformly distributed on the area of cross-section of the end face. Compatibility of longitudinal displacements of the shell and ribs is provided only at the points on the shell edges.

We specify the shell deflection in the form of the sum

$$w = w_1 + w_2 = f_1 \sin d_1 \xi \sin n_1 \theta + f_2 \sin d_2 \xi \sin n_2 \theta$$

where $f_1, f_2$ are varying parameters,

$$d_1 = \frac{\pi m}{\xi_1} \quad (s = 1, 2), \quad m_1 \quad \text{and} \quad m_2, \quad 2n_1 \quad \text{and} \quad 2n_2$$

are numbers of half-waves in the longitudinal and peripheral directions. The preliminary buckling forms correspond to the first summand of (5), and forms of the deflection of general stability loss correspond to the second summand.

After buckling of the shell in the form of $w_1 = f_1 \sin d_1 \xi \sin n_1 \theta$, the shell behavior with increasing of load is considered as behavior of a nonlinear system, and the expression for deflection doesn’t change with the increase of the amplitude. Stability loss conditions are of the form

$$\frac{\partial \Pi(w)}{\partial f_1} = 0, \quad \frac{\partial \Pi(w)}{\partial f_2} = 0$$

(6)

Using the strain compatibility equation (4), we find the stress function

$$\varphi = E \left\{ f_1 \left( \frac{n_1^2}{32} \cos 2d_1 \xi + \frac{d_1^2}{n_1^2} \cos 2n_1 \theta \right) - \frac{f_1^2}{32} \frac{d_1^2 r}{(d_1^2 + n_1^2)^2} \sin d_1 \xi \sin n_1 \theta - \frac{f_2}{2} \frac{d_2^2 r}{(d_2^2 + n_2^2)^2} \sin d_2 \xi \sin n_2 \theta - \frac{1}{4} f_1 f_2 \left[ \frac{(d_1^2 n_2 - d_2^2 n_2)^2}{(b_1^2 + b_2^2)^2} \cos b_1 \xi \cos b_2 \theta - \frac{(d_1^2 n_2 + d_2^2 n_2)^2}{(b_1^2 + b_2^2)^2} \cos b_1 \xi \cos b_2 \theta - \frac{(d_1^2 n_2 - d_2^2 n_2)^2}{(b_1^2 + b_2^2)^2} \cos b_1 \xi \cos b_2 \theta + \frac{(d_1^2 n_2 + d_2^2 n_2)^2}{(b_1^2 + b_2^2)^2} \cos b_1 \xi \cos b_2 \theta - \frac{r^2 \sigma_\theta^2}{2E} \right] \right\}$$

(7)

where

$$b_1 = n_2 - n_4, \quad b_2 = n_2 + n_4,$$

$$b_3 = d_1 - d_2 + b_4 = d_1 + d_2$$

After substitution of the expressions for $w$ and $\varphi$ into (1)-(3) we get

$$\Pi = \frac{\pi E h L_1}{48(1 - \nu^2)r^3} \left[ f_1^2 \left( \frac{d_1^2 + n_1^2}{2} \right)^2 + f_2^2 \left( \frac{d_2^2 + n_2^2}{2} \right)^2 \right] + \frac{\pi E h L_1}{2r^3} \left[ f_1^2 \left( \frac{d_1^2 + n_1^2}{2} \right)^2 \right] + \frac{f_2^2}{2} \left( \frac{d_2^2 + n_2^2}{2} \right)^2 + \frac{1}{2} f_2^2 \left( \frac{d_2^2 + n_2^2}{2} \right)^2$$

(8)
\[ A_4 = \frac{3}{4} \delta (1 - \nu^2) \left( d_m n_k - d_w n_1 \right)^4. \]
\[
+ \left[ \frac{1}{(b_1^2 + b_2^2)} + \frac{1}{(b_3^2 + b_4^2)} \right] \left( d_m n_2 + d_w n_1 \right) \left( d_m n_1^2 + d_w n_2 \right) \right] \right]
\]
\[ A_4 = -3(1 - \nu^2) \left( d_m^2 + d_w^2 \right) \pi(1) \sigma_1, \]
\[ A_4 = \left( d_m^2 + n_1^2 \right) \left( d_m^2 + n_2^2 \right) + \frac{2}{a^2} \gamma(1) \sigma_1 d_m^4 + \gamma(2) \sigma_1 n_1 d_m^4,
\]
\[ A_4 = -3(1 - \nu^2) \left( d_m^2 + d_w^2 \right) \pi(1) \sigma_1, \]
\[
\frac{\gamma(1) \sigma_1 d_m^4 + \gamma(2) \sigma_1 n_1 d_m^4}{A_4} = \frac{1}{2} - \frac{\sin 2d_m \xi_2}{4d_m}, \quad S_1 = \frac{1}{2} - \frac{\sin 2d_m \xi_2}{4d_m}
\]
\[
\frac{\gamma(1) \sigma_1 d_m^4 + \gamma(2) \sigma_1 n_1 d_m^4}{A_4} = \frac{1}{2} - \frac{\sin 2d_m \xi_2}{4d_m}, \quad S_1 = \frac{1}{2} - \frac{\sin 2d_m \xi_2}{4d_m}
\]
and relative deflection of the shell in the state preceding the general stability loss:
\[
\bar{\eta}_n^2 = -\frac{A_2 + A_6 + A_7 \eta}{2A_4}.
\]

It should be noted that when \( A_4 = A_6 = 0 \) equations (9) and (10) transform to the formulas given in [1] and corresponding to shells without medium. The parameter \( \eta \) should be compared with \( \eta_p \) that corresponds to the least critical load with the bending of longitudinal ribs without taking into account the shell buckling. While calculating the critical load parameter, the minimization of \( \eta \) is carried out with respect to the form parameters \( m \) and \( n \). The domain of each of these parameters is constructed in the vicinity of those values of \( m \) and \( n \) that correspond to the minimum of \( \eta_p \).

**IV. RESULTS AND CONCLUSIONS**

The results of the calculation of critical load parameter \( \eta \) taking into account the preliminary buckling of the shell are given in Table 1. The domain of reinforcement parameters is chosen so that the condition \( \eta_p = \alpha \eta_{\infty} \quad (\alpha \in [2;3]) \), \( \eta_{\infty} \) is critical stresses parameters of the shell) is satisfied. The following parameters are common for these shells: \( h^* = 1/140; \xi_2 = 2 \); the number of lateral ribs \( k_1 = 3 \); longitudinal ribs of cross-section with the ratio of height to width \( \psi_i = 14 \). Ratio of the area of lateral section of longitudinal ribs to the cross-section of the shell varies \( \psi(1) = 0.61 \). The number of longitudinal ribs also varies: \( k = 16, 24, 32 \). Annular ribs had the following characteristics:

\[
\frac{\eta_{\infty}^{(2)}}{a^2(k_i + 1)} = 8.4 \quad \frac{\mu^{(2)}}{a^2(k_i + 1)} = 0.054.
\]

The following values were accepted for the filler: \( \bar{\eta} / \bar{q}_o = 3 \); \( \bar{q}_o / E = 0.002 \).

As it is seen from Table 1, for the considered shells the ratio \( \eta / \eta_p \in [0.82;1] \). The ratio \( \eta / \eta_p \) increases with the growth of the number of longitudinal ribs at the same their total cross-section area. The numbers \( m \) and \( n \) coincide with the numbers \( m \) and \( n \) corresponding to \( \eta_p \) or differ insignificantly from them. The form of stability loss with the bending of ribs may respond to symmetric buckling with respect to the mean surface.
Table 1. Results of the calculation of critical load parameter $\eta$ taking into account the preliminary buckling of the shell

<table>
<thead>
<tr>
<th>$\gamma^{(i)}$</th>
<th>$k$</th>
<th>$\eta_0$</th>
<th>$\eta$</th>
<th>$\tilde{f}$</th>
<th>$\eta/\eta_0$</th>
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<tr>
<td>0.6</td>
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<td>1.701</td>
<td>1.751</td>
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REFERENCES

BIOGRAPHY
Ramiz (Aziz oglu) Iskanderov was born in Krasnoselo, Armenia in July 7, 1955. He received the M.Sc. degree in Mathematics-Mechanics from Azerbaijan (Baku) State University (Baku, Azerbaijan) in 1977. He also received the Ph.D. degree in Mathematics-Mechanics from Azerbaijan University of Architecture and Construction (Baku, Azerbaijan) in 1983. He is Associate Professor in the field of Mathematics-Mechanics in Department of Theoretical Mechanics, Azerbaijan University of Architecture and Construction (Baku, Azerbaijan) since 1983. He has published 41 papers and one book. His scientific interests are theoretical mechanics, solid state mechanics and mechanics of composite materials.