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ADVANCED METHOD TO SOLVE NODE EQUATIONS IN POWER SYSTEMS

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Abstract- Calculation of electrical system's steady-state regimes is practically carried out according to the node voltages equations and solved with the Zeydel's method, when equations are written as complex values. Software algorithm of electrical system's steady-state regimes developed for computers is based on solution of nonlinear node equations written as balance of powers or currents. Advanced method offered in the article enables to simplify algorithm of task solution, when systems generator nodes are set in parameters *P* and *U* and accelerate solution of node equations with iterative way in specific case.

Keywords: Electrical System's Steady-State Regimes, Generator Nodes, Node Voltages Equations, Zeydel's Method, Iterative Solution.

I. INTRODUCTION

In general case, electrical systems Node Voltages Equations (NVE) are nonlinear and express balance of currents or powers. Dependent on a choice of variables these equations are written in different ways:

- As complex voltages ($\dot{U}$), in this case each node of scheme is expressed by one equation with complex coefficient.
- As longitudinal and cross components of voltage, in this case each node of scheme is expressed by one pair of equation with real coefficient and equations are written on Cartesian coordinate system U', U''.
- As module and phase angle of voltages, in this case one pair of equation with real coefficient expresses each node of scheme and equations are written on polar coordinate system U, δ.

At calculation of system's steady-state regimes generator nodes are basically given in two kinds, with parameters *P*-*Q* or *P*-*U*. In first case, active and reactive power of generator is known and *P* = const, *Q* = const are accepted. The specified equations do not correspond to real control condition of system's regime, because, there is no regulator maintaining reactive power constant in generators. On the other hand, in a case if generator node represents one part of an electrical system, its reactive power can vary in very large limits, so what is difficult to determine beforehand. If generator node is given in parameters *P*-*U*, its active power and module of voltage are known and *P* = const, *U* = const are accepted [1].

These equations correspond to the real control condition of system and it becomes possible to maintain parameters *P* and *U* constant by regulating frequency and excitation in generator.

II. NODE VOLTAGES EQUATIONS

Let us assume that NVE of electrical system having *n* number of independent nodes are written down as complex voltages and express balance of currents [2, 3]:

$$\mathrm{Y}_{ii}\dot{U}_i+\sum_{\substack{j=1\\ j\neq i}}^{n} Y_{ij}\dot{U}_j=\frac{\widehat{S}_i}{\widehat{U}_i}-Y_{ib}\dot{U}_b \quad , \quad i=\overline{1,n} \qquad (1)$$

where, Y_{ij} is Eigen complex conductivity of node *I*, Y_{ij} is conductivity (with reverse sign) between nodes *i* and *j*, Y_{ib} is conductivity (with reverse sign) between nodes *i* and *b* (basic), $\dot{U}_i$, $\dot{U}_j$, $\dot{U}_b$ are complex voltage of independent nodes and basic node, $\dot{U}_b=U_b\angle 0^\circ$, $\widehat{S}_i$, $\widehat{U}_i$ are conjugate of complex total power and voltage of node *i*, $\widehat{U}_i=U_i\angle-\delta_i$, $\widehat{S}_i=P-jQ$, therefor we get:

$$Y_{ii}\dot{U}_i+\sum_{\substack{j=1\\ j\neq i}}^{n} Y_{ij}\dot{U}_j+Y_{ib}\dot{U}_b=\frac{P_i-jQ_i}{\widehat{U}_i} \quad , \quad i=\overline{1,n} \qquad (2)$$

where, P_i, Q_i are active and reactive power of node *i*.

Independent nodes can be applied to generator nodes, load nodes in system or any other nodes in scheme. Let's consider generator nodes. Generator node is considered separate power plants, large synchronous compensators connected to node point in scheme and equivalent source (generator) representing one part of electrical system. Four regime parameters participate in equation of these nodes, active power (*P*) and reactive power (*Q*) of generator, module of voltage in node (*U*) and phase angle (δ).

Two of them are accepted as independent variable and their value is known from initial conditions of a task. Usually parameters *P*, *Q* or *P*, *U* are accepted as independent variable, what means generator nodes are given out as *P*-*Q* or *P*-*U*. Then in first case units *U* and δ and in second case units *Q* and δ are determined from solution of NVE.

III. ZEYDEL'S METHOD

Solving the Equation (1) by the Zeydel's method with iterative way the equation of generator node i at iteration k is written thus:

$$Y_{ii}\dot{U}_i^{(k)}+\sum_{j=1}^{i-1}Y_{ij}\dot{U}_j^{(k)}+\sum_{j=i+1}^{n}Y_{ij}\dot{U}_j^{(k-1)}+Y_{ib}\dot{U}_b=\frac{P_i-jQ_i}{\hat{U}_i^{(k-1)}} \quad (3)$$

As it is known, in this case initially the complex value of voltage at first approximation ($\dot{U}^{(0)}=U^{(0)}\angle\delta^{(0)}$) is accepted, then solving the Equation (1) separately the value of voltage at next approximation is found out, etc. So the value of complex voltage of generator node at any iteration can be found by the formula:

$$\dot{U}_i^{(k)}=\frac{1}{Y_{ii}}\Big(\frac{P_i-jQ_i}{\hat{U}_i^{(k-1)}}-\sum_{j=1}^{i-1}Y_{ij}\dot{U}_j^{(k)}-\sum_{j=i+1}^{n}Y_{ij}\dot{U}_j^{(k-1)}-Y_{ib}\dot{U}_b\Big) \quad (4)$$

The shown order of calculation concerns the case when generator node is given out in parameters P-Q. However, if these nodes are given out in parameters P-U, because value of generator's reactive power is not known, it is not possible to use the Equation (4) directly and calculation becomes complicated. According to the existing order of calculation the values $\delta_i^{(k)}$ and $Q_i^{(k)}$ of generator node i given in parameters P_i-U_i at approximation k are determined in the below listed sequence [2, 4]:

1- In the Equation (4) accepting reactive power of generator node equal to the $Q_i = 0$ the initial value of voltage U_i at approximation k is calculated:

$$V_i^{(k)}=\frac{1}{Y_{ii}}\left(\frac{P_i}{\hat{U}_i^{(k-1)}}-\dot{B}_i^{(k)}\right)=V_i'^{(k)}+jV_i''^{(k)} \quad (5)$$

$$\dot{B}_i^{(k)}=\sum_{j=1}^{i-1}Y_{ij}\dot{U}_j^{(k)}+\sum_{j=i+1}^{n}Y_{ij}\dot{U}_j^{(k-1)}+Y_{ib}\dot{U}_b=B_i^{(k)}\angle\beta_i^{(k)} \quad (6)$$

2- The intermediate value X is entered in to the calculation and its value is determined:

$$X=V_i'^{(k)}\left(b_{ii}U_i'^{(k-1)}+g_{ii}U_i''^{(k-1)}\right)-V_1''^{(k)}\left(g_{ii}U_i'^{(k-1)}-b_{ii}U_i''^{(k-1)}\right) \quad (7)$$

3- $Q_i^{(k)}$ is determined from solution of quadratic equation written relative to reactive power of node i:

$$Q_i^{(k)}=-X\pm\sqrt{X^2-\left(V_i^{(k)2}-U_i^{(k)2}\right)y_{ii}^2U_i^{(k-1)2}} \quad (8)$$

4- The value of complex voltage of node i is found out:

$$\dot{U}_i^{(k)}=\dot{V}_i^{(k)}-j\frac{Q_i^{(k)}}{Y_{ii}\hat{U}_i^{(k-1)}} \quad (9)$$

Thus, solution of the generator nodes equation given in parameters P-U by the Zeydel's method is carried out with the Equation (5) to Equation (9) instead of Equation (4), which is relatively complicated.

IV. PROPOSED METHOD FOR SIMPLIFICATION OF EQUATIONS SOLUTION ANALYTICAL COMPARISON

With the purpose of easing the calculation, the following simplifying method is offered. Taking into account the Equation (6) let us write the Equation (4) in this way:

$$Y_{ii}\dot{U}_i^{(k)}+\dot{B}_i^{(k)}=\frac{P_i-jQ_i^{(k)}}{\hat{U}_i^{(k-1)}} \quad (10)$$

From here,

$$Y_{ii}\dot{U}_i^{(k)}\hat{U}_i^{(k-1)}+\dot{B}_i^{(k)}\hat{U}_i^{(k-1)}=P_i-jQ_i^{(k)} \quad (11)$$

or if we write,

$$Y_{ii}=y_{ii}\angle-\varphi_{ii} \quad (12)$$

$$\dot{U}_i^{(k)}=U_i^{(k)}\angle\delta_i^{(k)} \quad (13)$$

$$\hat{U}_i^{(k-1)}=U_i^{(k-1)}\angle-\delta_i^{(k-1)} \quad (14)$$

$$\dot{B}_i^{(k)}=B_i^{(k)}\angle\beta_i^{(k)} \quad (15)$$

and also taking into account $U_i^{(k)}=U_i^{(k)}=U_i$ for module of voltage we get,

$$y_{ii}U_i^2\angle\left(\delta_i^{(k)}-\delta_i^{(k-1)}-\varphi_{ii}\right)+B_i^{(k)}U_i\angle\left(\beta_i^{(k)}-\delta_i^{(k-1)}\right)=P_i-jQ_i^{(k)} \quad (16)$$

If split up the left side of the last expression in to the real and imaginary parts and equal them accordingly to the P_i and $Q_i^{(k)}$, we get the below two equations:

$$y_{ii}U_i^2\cos\left(\delta_i^{(k)}-\delta_i^{(k-1)}-\varphi_{ii}\right)+B_i^{(k)}U_i\cos\left(\beta_i^{(k)}-\delta_i^{(k-1)}\right)=P_i \quad (17)$$

$$y_{ii}U_i^2\sin\left(\delta_i^{(k)}-\delta_i^{(k-1)}-\varphi_{ii}\right)+B_i^{(k)}U_i\sin\left(\beta_i^{(k)}-\delta_i^{(k-1)}\right)=-Q_i^{(k)} \quad (18)$$

where, are two unknowns as $\delta_i^{(k)}$ and $Q_i^{(k)}$. The value of $\delta_i^{(k)}$ is found out from the Equation (17):

$$\cos\left(\delta_i^{(k)}-\delta_i^{(k)}-\varphi_{ii}\right)=\frac{P_i-B_i^{(k)}U_i\cos\left(\beta_i^{(k)}-\delta_i^{(k-1)}\right)}{y_{ii}U_i^2} \quad (19)$$

Then reactive power of generator node at iteration k is calculated with the Equation (19). But as the value of $\delta_i^{(k)}$ is already known, there is no need to find out the power $Q_i^{(k)}$, to calculate this power matters only at the end of iteration process.

Thus with offered method the only two Equations (6) and (19) are used at determination of the phase angle value of the generator node i voltage at iteration k, so what is considerably simple. The block scheme of the calculation algorithm is specified in the Figure 1, where ε is desirable accuracy of calculation.

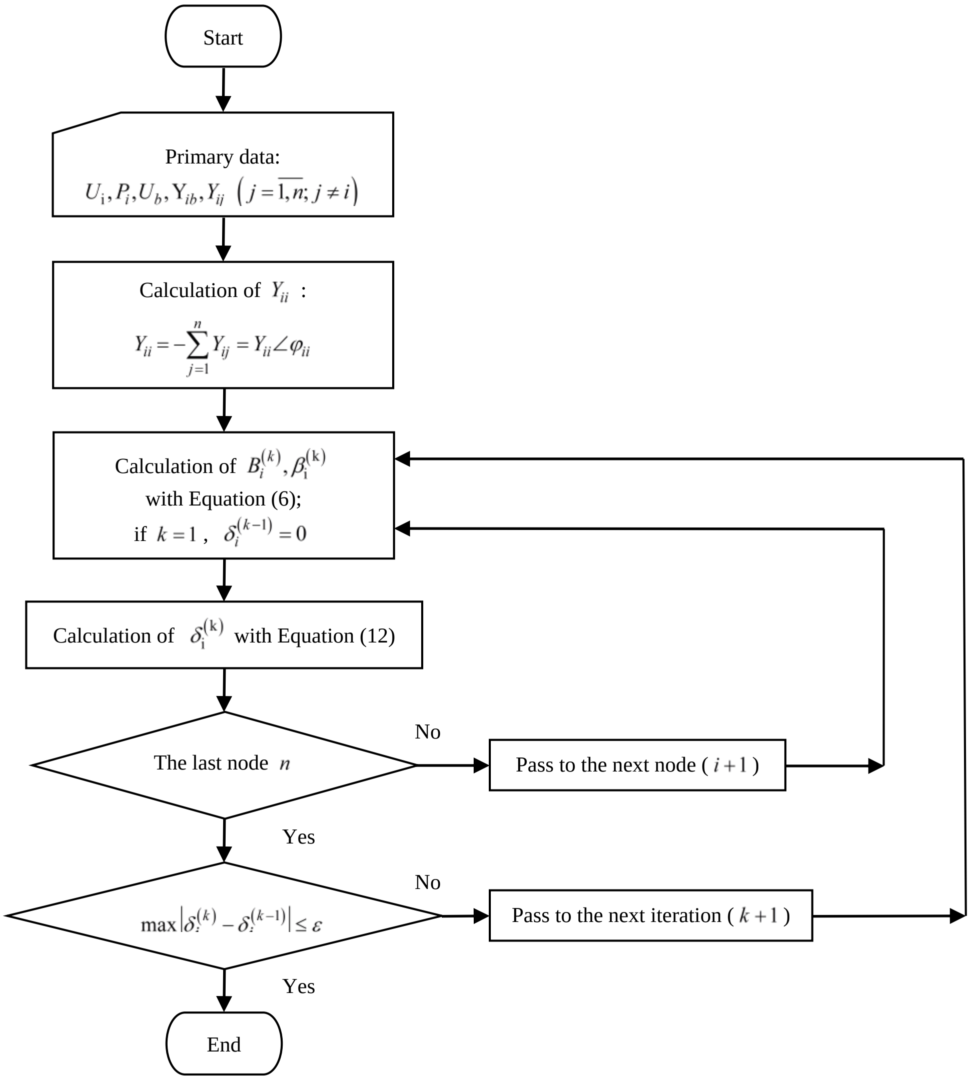

Figure 1. The block scheme of the calculation algorithm

V. ACCELERATION OF ITERATIVE PROCESS

Although the method above allows finding out the voltage $\dot{U}_i^{(k)}$ with a simple way, but it does not change number of iterations in calculation. Using the advanced method described below the solution of the Equation (4) is not only even much more simplified, the iterative solution of NVE becomes accelerated too. As may be seen from the Equation (3), here two different values of complex voltage in considered node *i* are participated, $\dot{U}_i^{(k-1)}$ in the right side and $\dot{U}_i^{(k)}$ in the left side.

In subsequent iterations difference between these units gradually decreases and it becomes $\dot{U}_i^{(m)} \approx \dot{U}_i^{(m-1)}$ at last iteration *m*. It gives a base to state that, if $\dot{U}_i^{(k)}$ is written instead of $\dot{U}_i^{(k-1)}$ in the right side of the Equation (2), then the found value for $\dot{U}_i^{(k)}$ from the solution of this equation will be more closer to true (exact) value of a voltage, that means acceleration of iterative process. Indeed value of $\dot{U}_i^{(k)}$ depends on under Equation (4).

Therefore replacing voltage $\dot{U}_i^{(k-1)}$ with $\dot{U}_i^{(k)}$, which is exacter than it, the value of $\dot{U}_i^{(k)}$ calculated on the Equation (4) will turn out much exacter. This conclusion is also compatible to process of steady-state regime establishment in electrical systems [5].

VI. SIMULATION RESULTS AND ANALYTICAL CALCULATIONS

The effectiveness of the proposed method is verified practically by observing a sequence of steady-state regime establishment on the Calculation Desk (CD) of an alternating current, what is mathematical model of electrical system.

Thus, system's steady-state regime is obtained by adjusting voltage and phase angle of separate model generators iteratively in several cycles by turns in scheme of electrical system typed in CD. Here each cycle of regulation corresponds to one iteration step of analytical calculation and such conformity is satisfied only if $\dot{U}_i^{(k-1)} \approx \dot{U}_i^{(k)}$ is accepted in the Equation (4). Thus having written $\dot{U}_i^{(k-1)} \approx \dot{U}_i^{(k)}$ in the Equation (2) we receive:

$$Y_{ii}\dot{U}_i^{(k)} + \dot{A}_i^{(k)} = \frac{P_i^{(k)} - jQ_i^{(k)}}{\hat{U}_i^{(k)}} \tag{20}$$

where, value of the complex unit $\dot{A}_i^{(k)}$ is calculated on the follows expression:

$$\dot{A}_i^{(k)} = \sum_{j=1}^{i-1} Y_{ij}\dot{U}_j^{(k)} + \sum_{j=i+1}^{n} Y_{ij}\dot{U}_j^{(k-1)} + Y_{ib}\dot{U}_b \tag{21}$$

From the Equation (20) we can write:

$$Y_{ii}\dot{U}_i^{(k)2} + \hat{U}_i^{(k)} A_i^{(k)} = P_i^{(k)} - jQ_i^{(k)} \tag{22}$$

Let us write complex units in Equation (22) as in power:

$$\dot{U}_i^{(k)} = \dot{U}_i^{(k)} \angle \delta_i^{(k)} \tag{23}$$

$$\hat{U}_i^{(k)} = \dot{U}_i^{(k)} \angle -\delta_i^{(k)} \tag{24}$$

$$\dot{A}_i^{(k)} = \dot{A}_i^{(k)} \angle \alpha_i^{(k)} \tag{25}$$

$$Y_{ii} = Y_{ii} \angle -\varphi_{ii} = g_{ii} - jb_{ii} \tag{26}$$

Then the Equation (15) will turn out so:

$$\begin{aligned} &g_{ii}U_i^2 + U_i A_i^{(k)} \cos(-\delta_i^{(k)} + \alpha_i^{(k)}) - \\ &-j\left[b_{ii}U_i^2 - U_i A_i^{(k)} \sin(-\delta_i^{(k)} + \alpha_i^{(k)})\right] = P - jQ \end{aligned} \tag{27}$$

From here, we get:

$$g_{ii}U_i^2 + U_i A_i^{(k)} \cos(-\delta_i^{(k)} + \alpha_i^{(k)}) = P_i \tag{28}$$

$$b_{ii}U_i^2 - U_i A_i^{(k)} \sin(-\delta_i^{(k)} + \alpha_i^{(k)}) = Q_i \tag{29}$$

Now let's consider use of the Equations (28) and (29). The units P_i and U_i are known if generator node is given out as *P-U*. In this case, δ_i is found out from solution of the Equation (28):

$$\cos(-\delta_i + \alpha_i) = \frac{P_i - g_{ii}U_i^2}{U_i A_i} \tag{30}$$

Then Q_i is calculated from the Equation (29). Let us note that in the considered case there is no need to determine reactive power at separate iterations, it is reasonable to know its value only at the end of iteration process.

VII. CONCLUSIONS

Analyzing the solutions of electrical systems node voltages equations for steady-state regimes calculation, the equations are written down as a balance of currents and with complex coefficients. Considered the complexities arising in a case when generator nodes are given with parameters *P* and *U*. Proposed advanced method, which enables to simplify and to speed up the solution of generator nodes nonlinear equations by Zeydel's method at electrical system's steady-state regime's calculation. When generator nodes are given in *P-U* form and nonlinear equations are written down with complex coefficients.

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BIOGRAPHY

Laman Epikur Suleymanli was born in Baku, Azerbaijan, 1974. She graduated from Azerbaijan State Oil Academy and was conferred the qualification of Systems Engineer, specializing in Automated Data Processing and Control Systems. Further, she graduated from British Petroleum Caspian Technical Training Centre and specialized as Electrician in Oil and Gas Industry. She has 16 years' experience of Electrical and Instrument & Control Engineer working in "Azerenerji" JSC, Azerbaijan State Oil Academy, Baku Electric Network Ltd all in Baku, Azerbaijan. She was scientist responsible for Optimization of Electrical Systems in Azerbaijan Power Research Institute (Baku, Azerbaijan) and Electrical Supervisor in British Petroleum's PCWU project. Currently she is working in British Petroleum Exploration Ltd, Safety & Operational Risk, and Engineering Authority & Technical Support Team, responsible for Process & Instrument, Control, and Electrical division. She is doing scientific-research work on static stability of electrical systems loads, which reflected in tens of articles published both in national and international journals and in conferences. She is member of Azerbaijan Post-Graduate Students and Young Researchers Association.