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MODELING OF MECHANICAL STRESS PROCESS OF TRANSFORMER WINDING CAUSED BY SHORT CIRCUIT CURRENTS

V.A. Lavrinovich Y.N. Isaev A.V. Mytnikov

National Research Tomsk Polytechnic University, Tomsk, Russia
lavr_work@tpu.ru, isaev_yusup@mail.ru, mytnikov66@mail.ru

Abstract- This paper is focused on modeling of electromagnetic process in transformer windings which are caused by external fault short-circuit currents. Detailed considerations of axial and radial shifts as well as difficult winding movement are given. The simulations are based on relations allow to estimate a winding shift at the known current. Calculations of ponderomotive force distribution for the windings of three-phase power transformer model are carried out. Process and result of short-circuit currents influence are given. Example of ponderomotive force calculation for circle and rectangular section of transformer winding is discussed.

Keywords: Transformer Windings, Transformer Diagnostics, Longitudinal and Transverse Deformations, Distributed Capacitance, Ponderomotive Forces, Mechanical Stress, Maxwell's Equations, Maxwell's Stress Tensor.

I. INTRODUCTION

Large power transformers belong to the most expensive and strategically important components of any power generation and transmission system. A serious failure of a large power transformer due to insulation breakdown can generate substantial costs for repair and financial losses due to power outage. Therefore, utilities have clear incentive to assess the actual condition of their transformer, in particular the condition of the high voltage (HV) insulation system, with the aim to minimize the risk of failures and to avoid forced outages of strategically important units. One major reason for internal faults is result of electromagnetic forces impact. So prediction of winding behavior under short-circuit current influence is important task.

There are a lot of different models and approaches to this question are developed in high voltage laboratories with detailed description [1-6]. We consider this problem in connection with of winding diagnostics procedure. We have being developing winding state control technology using short probe pulse (nanosecond range). It is based on "classical" pulsed technique which was proposed in [7]. Our approach implies probe pulse with rapid front and 300-400 ns duration, which increases a sensitivity of whole diagnostics procedure [8].

Goal of this work is make calculations of ponderomotive force distribution for the windings of three-phase power transformer model taking into account diagnostics procedure. It is necessary to note that all calculations and results below are original, received by authors and published never before.

II. THE CALCULATION OF PONDEROMOTIVE FORCES

Electrodynamic firmness is rapidly reduced at short-circuit current flow. Result of this is mechanical stresses, such as axial and radial shifts, twisting and untwisting of windings and its defect combination. In "nanosecond pulsed technology" probe pulse has frequency range with very rich high frequency spectrum. High-frequency components of probe pulse excite bias currents, which in turn, flow through distributed capacities of transformer form unique spectrum of output signal.

Winding defects caused by shift or mechanical stress lead to changes of distributed capacities and output spectrum content. So, comparison of two output spectrum, one of them is for winding without defect ("healthy transformer") and other which is for defect winding allows stresses presence [9]. So, all mechanical shift and deformations change meanings of distributed capacities of transformer. Therefore one of goal is main formulas obtain for estimation of winding stress degree caused by short circuit currents.

At first, make calculations for axial and radial shifts of one winding turn at the famous current. Then, carry out calculations of field distribution in separate winding. A change of volume body is characterized by components of displacement vector du_i which are determined by deformation tensor:

$$du_i = \varepsilon_{ij} dx_j \tag{1}$$

where summation is fulfilled with dummy subscripts. We propose that is not a rotation deformation. It is possible bring a deformation tensor to main axes. It means that tensor has diagonal elements only due to axes orientation.

$$\begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix} \tag{2}$$

The diagonal elements are called "main deformations" and describe expansions and compressions of volume element in main axes direction. Tensor trace ε_{ii} is sum of diagonal elements and relative elongation of volume, which is result with Equation (1) accounting [10]:

$$\frac{\Delta V}{V} = \varepsilon_{ii} = \frac{\partial u_i}{\partial x_i} = \mathrm{div}(u) \tag{3}$$

Between tensor components is connection:

$$\varepsilon_{33} = \frac{\partial u}{\partial z} = \frac{\sigma_{33}}{E} \tag{4}$$

where σ_{33} is element of tensor along axis z and E is Young modulus (expansion modulus),

$$\varepsilon_{11} = \frac{\partial u}{\partial x} = -\sigma\varepsilon_{33} = \sigma\frac{\partial u}{\partial z} \tag{5}$$

σ is Poisson ratio (ratio of lateral contraction to longitudinal extension).

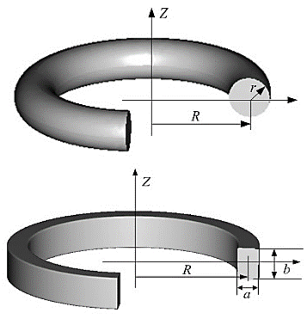


Figure 1. Geometrical parameters of winding turns: upper turn is for circular section; down turn is for rectangular section

Further it is necessary to determine a deformation of copper wire with magnetic permeability μ=1 under influence of magnetic field. Let write that $\sigma_{\|}$ is mechanical stress along wire axis, $\sigma_{\perp}$ is mechanical stress across axis. Next step is these stresses determination. Force $F_{\|}$ is lateral stress $\sigma_{\|}$ which is multiplied to square cross-section $\sigma_{\|}2r^2$ and taking into account changing of magnetic field energy, it could be described:

$$F_{\|} = \sigma_{\|}\pi r^2 = \frac{\partial W}{\partial l} = \frac{I^2}{2}\times\frac{\partial L}{\partial l} = \frac{I^2}{4\pi}\times\frac{\partial L}{\partial R} \tag{6}$$

where l=2πR as wire length, L is ring inductance and I is wire current. Force $F_{\perp}$ is acting across wire axis is longitudinal stress $\sigma_{\perp}$ which is multiplied to surface square of wire and taking into account changing of magnetic field energy along radius r, it could be described as:

$$F_{\perp} = \sigma_{\perp}2\pi rR = \frac{\partial W}{\partial r} = \frac{I^2}{2}\times\frac{\partial L}{\partial r} \tag{7}$$

Equations (6) and (7) allow to determine connection between longitudinal and lateral stresses:

$$\begin{aligned} \sigma_{\|} &= \frac{F_{\|}}{\pi r^2} = \frac{I^2}{4\pi^2 r^2}\times\frac{\partial L}{\partial R} \\ \sigma_{\perp} &= \frac{F_{\perp}}{2\pi rR} = \frac{I^2}{4\pi rR}\times\frac{\partial L}{\partial r} \end{aligned} \tag{8}$$

Now write an expression for wire inductance in ring form with circular cross section:

$$L(r,R) = \mu_0 .R\times\left(\ln\left(\frac{8R}{r}\right)-\frac{7}{4}\right) \tag{9}$$

Using the Equations (8) and (9):

$$\begin{aligned} \sigma_{\|} &= \frac{I^2\mu_0}{4\pi^2 r^2}\times\left(\ln\left(\frac{8R}{r}\right)-\frac{3}{4}\right) \\ \sigma_{\perp} &= -\frac{2I^2\mu_0}{\pi r^2} \end{aligned} \tag{10}$$

Taking into account Equations (4) and (5), it is possible to write expression for relative elongation of wire with circular cross section:

$$\begin{aligned} \frac{\Delta l}{l} &= \frac{1}{E}(\sigma_{\|}-2\sigma\sigma_{\perp}) = \\ &= \frac{I^2\mu_0}{E\pi r^2}\times\left(\frac{1}{4\pi}\times\left(\ln\left(\frac{8R}{r}\right)-\frac{3}{4}\right)+4\sigma\right) \end{aligned} \tag{11}$$

When we make the same operations for wire with rectangular cross section, expression for inductance looks like:

$$L(r,R) = \mu_0 R\left(\ln\left(\frac{8R}{b+a}\right)-\frac{1}{2}\right) \tag{11}$$

$$\frac{\partial L}{\partial R} = \mu_0\left(\ln\left(\frac{8R}{b+a}\right)+\frac{1}{2}\right) \tag{12}$$

$$\frac{\partial L}{\partial a} = -\mu_0\frac{R}{b+a},\quad \frac{\partial L}{\partial b} = -\mu_0\frac{R}{b+a} \tag{13}$$

Stresses along and normal to axis could be expressed as:

$$\begin{cases} F_{\|} = \sigma_{\|}ab = \dfrac{\partial W}{\partial l} = \dfrac{I^2}{2}\times\dfrac{\partial L}{\partial l} = \dfrac{I^2}{4\pi}\times\dfrac{\partial L}{\partial R} \\ \Rightarrow \sigma_{\|} = \dfrac{I^2\mu_0}{4\pi ab}\times\left(\ln\left(\dfrac{8R}{a+b}\right)+\dfrac{1}{2}\right) \end{cases} \tag{14}$$

$$\begin{cases} F_{\perp 1} = \sigma_{\perp 1}2\pi Ra = \dfrac{\partial W}{\partial a} = \dfrac{I^2}{2}\times\dfrac{\partial L}{\partial a} \\ \Rightarrow \sigma_{\perp 1} = \dfrac{-\mu_0 I^2}{4\pi(b+a)a} \end{cases} \tag{15}$$

$$\begin{cases} F_{\perp 2} = \sigma_{\perp 2}2\pi Rb = \dfrac{\partial W}{\partial b} = \dfrac{I^2}{2}\dfrac{\partial L}{\partial b} \\ \Rightarrow \sigma_{\perp 2} = \dfrac{-\mu_0 I^2}{4\pi(b+a)b} \end{cases} \tag{16}$$

Relative elongation of ring is:

$$\begin{aligned} &\frac{\Delta l}{l} = \frac{1}{E}\times\left(\sigma_{\|}-\sigma\times\left(\sigma_{\perp 1}+\sigma_{\perp 2}\right)\right) = \\ &\frac{I^2\mu_0}{E\pi 4}\times\left(\frac{1}{ab}\times\left(\ln\left(\frac{8R}{a+b}\right)+\frac{1}{2}\right)-\frac{\sigma}{ab}\right) \end{aligned} \tag{17}$$

To find change of capacitance of wire of circular cross-section following formula is appropriate:

$$C(r,R,\varepsilon) = 4\pi\varepsilon R\times\left(0.68+1.07\times\frac{r}{R}\right) \tag{18}$$

Thermal deformation of capacity change could be neglected:

$$dC(r,R,\varepsilon)=\frac{\partial C}{\partial r}\times dr+\frac{\partial C}{\partial R}\times dR= \\ =\pi\varepsilon\times\left(4.28dr+2.27dR\right) \quad (19)$$

where, $dR=\frac{\sigma_{\|}}{E}R$ and $dr=\frac{2\sigma\sigma_{\perp}r}{E}$.

So, necessary expressions for estimation of axial and radial shifts of one winding turn and capacity change of one are obtained. To receive whole space distribution of deformations of transformer winding, it is necessary to take into account not only one turn inductance, but mutual inductance of all turns. Space distribution of deforming forces in electromagnetic field is called ponderomotive forces [10]. To right estimate the winding deformations, it should be calculated ponderomotive force distribution. To carry out this, system of Maxwell equations and Maxwell tension tensor should be used. Due to these, vector magnetic potential $\boldsymbol{A}$ and magnetic intensity $\boldsymbol{H}$ could be determined as:

$$\boldsymbol{H}=\nabla\times\boldsymbol{A}=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}= \\ \boldsymbol{i}\left(\frac{\partial}{\partial y}A_z-\frac{\partial}{\partial z}A_y\right)+\boldsymbol{j}\left(\frac{\partial}{\partial z}A_x-\frac{\partial}{\partial x}A_z\right) \\ +\boldsymbol{k}\left(\frac{\partial}{\partial x}A_y-\frac{\partial}{\partial y}A_y\right) \quad (20)$$

$$\begin{cases}\nabla\left(\mu\boldsymbol{H}\right)=\boldsymbol{J}\\ \Rightarrow\frac{\partial}{\partial x}\mu\frac{\partial}{\partial x}\boldsymbol{A}+\frac{\partial}{\partial y}\mu\frac{\partial}{\partial y}\boldsymbol{A}+\frac{\partial}{\partial z}\mu\frac{\partial}{\partial z}\boldsymbol{A}=\boldsymbol{J}\end{cases} \quad (21)$$

where, μ is magnetic permeability, $\boldsymbol{J}=(J_x, J_y, J_z)^T$ is current density of wire, and $\boldsymbol{A}=(A_x, A_y, A_z)^T$ is vector magnetic potential.

Vector magnetic potential $\boldsymbol{A}$ and next magnetic field intensity determination $\boldsymbol{H}$ could be obtained by Poisson equation decision. To receive distribution of Maxwell's tensor deformation, next expression is used:

$$\begin{cases}\boldsymbol{\sigma}_{jk}=\frac{\varepsilon}{4\pi}\left(H_iH_k-\frac{H^2\delta_{jk}}{2}\right)\\ \Rightarrow\frac{\varepsilon}{4\pi}\begin{pmatrix}H_x^2-\frac{H^2}{2} & H_xH_y & H_xH_z\\ H_yH_x & H_y^2-\frac{H^2}{2} & H_yH_z\\ H_zH_x & H_zH_y & H_z^2-\frac{H^2}{2}\end{pmatrix}\end{cases} \quad (22)$$

where, ε is dielectric permeability, H_i, (i=1, 2, 3) is component of magnetic field, $\boldsymbol{\sigma}=\sigma_{i,j}$, (i,j=1, 2, 3) is elements of mechanical stress tensor, and $\delta_{jk}=\begin{cases}1 & if \quad j=k\\ 0 & if \quad j\neq k\end{cases}$ is Kroneker symbol.

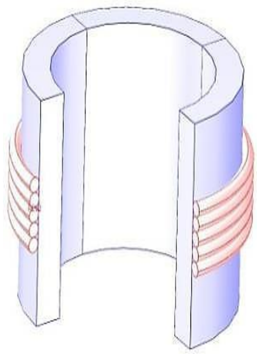
Integration of mechanical stress tensor along wire surface of winding allow to write the distribution of surface forces of winding deformation:

$$\boldsymbol{F}=\oint_S\boldsymbol{\sigma}dS=\oint_S\left(\mu\boldsymbol{H}(\boldsymbol{Hn})-\mu\boldsymbol{n}\frac{H^2}{2}\right)dS \quad (23)$$

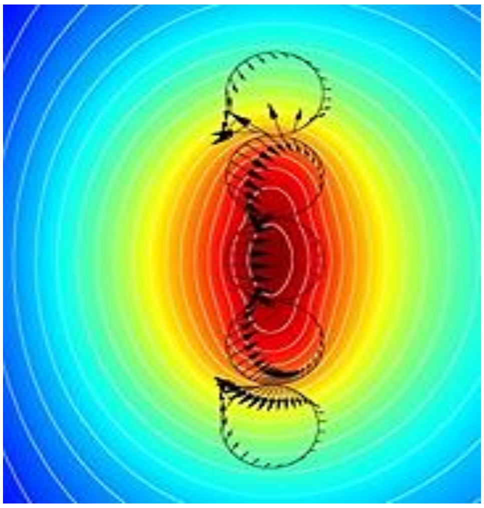
As example of calculation it is shown task decision of graph of distribution of surface forces in magnetic field by means of finite elements method (FEM). To do that software package Comsol Multiphysics is used.

Two separate windings are considered. One is reeled up by copper wire with diameter 2 mm and step 4 mm, turn number 5, inner diameter of winding base (polymeric cylinder) is 140 mm, outer diameter of one is 160 mm, copper wire length is 370 mm. Other winding is fulfilled by copper wire with rectangular cross section with geometrical sizes 4 and 7 mm respectively, turn number 5, inner diameter is 86 mm, outer diameter is 102 mm, common length is 370 mm.

Distribution of surface forces in turns of circular and rectangular cross section is shown on Figures 2 and 3. It is modeled that in three wires in center of winding, short circuit is occurred. Current value is high enough in these wires. It is could see that in short circuit place, strong magnetic field is excited. Resulting vectors show directions of wire displacements.

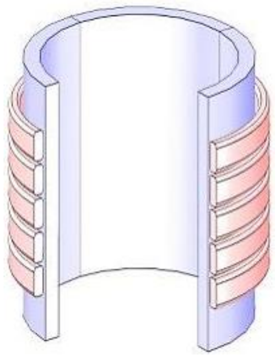


(a)

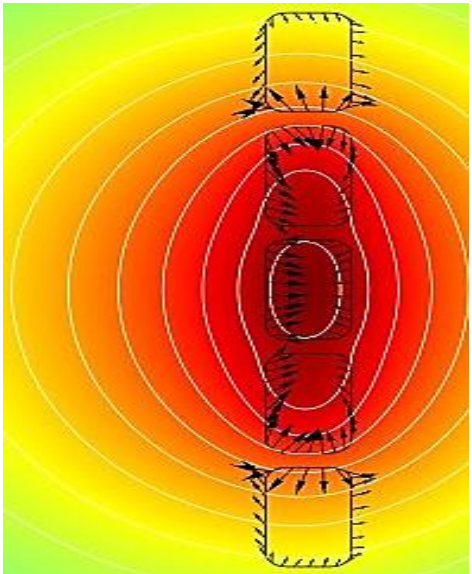


(b)

Figure 2. Distribution of surface forces in turns of circular cross section: (a) Coil part view
(b) Result of calculation in graphic presentation



(a)



(b)

Figure 3. Distribution of surface forces in turns of rectangular cross section: (a) coil part view
(b) result of calculation in graphic presentation

III. CONCLUSIONS

In presented paper expressions for estimation of axial and radial shifts have been obtained. Formulas for estimation of capacity change due to winding deformation have been received. Calculation of deformations showing the distribution of ponderomotive forces on the wire surface of transformer winding has been given. Resulting vectors of forces acting on separate turns at the short circuit currents have been shown.

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BIOGRAPHIES

Valeriy A. Lavrinovich was born on 7 November 1948. He received the Ph.D. degree from High Current Electronics Institute of Russian Academy of Sciences (Tomsk, Russia) in 1985, and Doctor of Science degree from the Tomsk Polytechnic University (Tomsk, Russia), in 2005. Both dissertations are dedicated to investigation and elaboration of high voltage vacuum spark gaps and commutators. He was director of Russian Research Institute of Electro Technical Industry from 1989 to 1998. Now he is Professor of Department of Electrical Power Engineering Systems and High Voltage Engineering. He is laureate of Russia Government Prize in area of Science and Technology. He published more than 90 scientific articles. His areas of interest include pulsed power and electrical power engineering.

Yusup N. Isaev was born on 20 March 1960. He received the Ph.D. degree in 1994. He defended dissertation on Doctor of Science degree in 2004. He is Professor of Tomsk of Department of Electrical Grids of Tomsk Polytechnical University (Tomsk, Russia). He is famous scientist in such area as electro and plasma dynamics, electric power engineering, numerical methods and modeling.

Alexey V. Mytnikov was born on 2 June 1966. He received Ph.D. degree in 2001. He is Associate Professor of Electric Energy System Department, Power Engineering Institute of Tomsk Polytechnical University (Tomsk, Russia). His areas of scientific interests are power transformer diagnostics, high voltage technologies.