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PULSATIONAL MOTION OF A MIXTURE IN A VIBRATING MEDIUM WITH CONSIDER FLOW RATE

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Abstract- It is known that the effects of non-conductivity substantially complicate the research and are most fully manifested when considering waves arising from the impact of shock and vibration loads [1]. These are liquids with bubbles. Knowledge of the regularities of the processes occurring in such continuous environ is of great importance for the creation of scientific bases for mathematical analysis. In this regard, the pulsed motion of two-phase adhesive fluid bubble enclosed in a viscous stretch tube is investigated. It is assumed that the cylinder is tightly connected to the environment. In the case of a limited length, the pressure is applied at one end of it. Therefore, these well-known and obvious works of this kind are generalized and developed. In a numerical experiment, we consider an infinite half-pipe with a flow of two-phase current flow: glycerin, water and oil containing small admixtures of air bubbles, respectively.

Keywords: Bubble Liquid, Viscous Elasticity, Waves, Pulsating Flow, Hidrodinamic Pressure, Flow Rate.

I. INTRODUCTION

The wave propagation phenomenon is very popular in the unstable tray with fluid flow in the cavity. Due to the problems of this type, it should be encouraged to consider the equivalent, given the fluid movement in it. A suitable one-dimensional approximation is expected when the pipe length is much larger than its radius. Such an approximation essentially describes the properties of "liquid coating". So far, the sum of such a problem, the dynamics of all flows is well developed. However, the mechanism of the phenomenon observed in relation to the two phase fluid in the chamber, due to the density, viscosity and tube orthosis, is not well understood.

The interest in fluid dynamics problems in the pipe alteration process has been given to the importance of using the research results to problems in the calculation of hydraulic systems in the fields of aviation, oil and gas, chemical technology and thermal dynamics [3]. Liquid and structural problems (FSIs) are generated by turbomachinery and industrial water pipeline explosion systems.

This often includes gas bubbles that greatly change the fluid dynamics [4]. The dynamic loading of fluid-filled flexible pipes was greatly considered as a problem, the FSI model was studied.

II. BASIC RELATIONS OF THE PROBLEM

Closed system of equations consists of hydro equations of fluid motion and tubes, as well as the equations components' velocity continuity on the border of the interface of the liquid and tube.

Biphasic mediums consisting of a mixture of liquid with tiny bubbles of gas are very important example of the relaxing environment. Experimental and theoretical studies have shown that when solving the problem of transporting of the two-phase liquid-gas flows, it should be kept in mind that such environments are different from other two-phase media [5]. The difference is that the heat of the carrier phase is much higher than the heat capacity of the dispersed phase due to the prevailing mass content of the carrier phase in the volume unit.

Therefore, the liquid can be regarded as a thermostat with constant temperature. Methods of continuum mechanics have been used in the basis of the theory used to describe the flows of bubbly mixtures. We establish the following hypotheses and assumptions that greatly simplify the formulation and solution of the problem, without altering the essence of the phenomenon:

- Bubbles are present in the form of spherical inclusions of the same radius r_0 in every elementary macro-volume. Furthermore, the volume of concentration of bubbles α_{20} is low (a mixture of mono - disperse) and the value r_0 is much smaller than the characteristic size of the problem;
- Direct interaction and collision of bubbles with each other can be ignored;
- Merge processes (coagulation), fragmentation and formation of new bubbles are absent;
- Velocity of the bubbles and carrier phases are the same;
- Bubbles have neutral buoyancy, i.e., do not settle down and do not float up;
- Viscosity of the carrier phase is much greater than the viscosity of the gas bubbles (such as the viscosity of water is 10 times greater than the viscosity of air) and, therefore the viscosity of the mixture practically does not depend on the volume fraction of bubbles.

III. THEORETICAL BACKGROUND

Within the given assumptions the hydrodynamic equations consist of the impulse equation

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0 \tag{1}$$

The equations of continuity [2]

$$\frac{2}{R}\frac{\partial w}{\partial t} + \frac{\partial u}{\partial x} + \frac{1}{\rho_0}\frac{\partial \rho}{\partial t} = 0 \tag{2}$$

and rheological equations of the mixture [1]

$$\frac{\xi}{\rho_0}\frac{\partial \rho}{\partial t} + a^2 \rho = p \tag{3}$$

In Equations (1)-(3), $u(x,t)$ is the rate of mixture flow, $p(x,t)$ is a hydrodynamic pressure and $\rho(x,t)$ is density of the mixture.

$$a^2 = \frac{1}{\alpha_{20}(1-\alpha_{20})} \cdot \left(\frac{\rho_{10}}{\rho_{10}-\rho_{20}}\right)^2 \cdot \frac{p}{\rho_{10}} \tag{4}$$

Square of the equilibrium sound speed

$$\rho = (1-\alpha_{20}) \cdot \rho_{10} + \alpha_{20} \cdot \rho_{20} \tag{5}$$

and

$$\xi = \frac{4}{3}\mu \frac{\alpha_{10}}{\alpha_{20}} \tag{6}$$

The volume viscosity μ is the dynamic viscosity of the carrier phase, α_{20} is the volume content of bubbles, ρ_{10}, ρ_{20} are the densities of the carrier and dispersed phase, respectively and p_0 is given static pressure. Subscript 0 indicates the value at the equilibrium phase. It should be noted that in the linear formulation the equilibrium α_{20} is used instead of the current volume concentration α_2, and this approach implies the presence of bubbles $(\alpha_{20} \neq 0)$. If the volume fraction of bubbles is sufficiently low $(\alpha_{20} << 1)$, then the medium can be considered as homogeneous. Specifics of such fluid is that when $\rho_{20} << \rho_{10}$, then

$$\rho = (1-\alpha_{20}) \cdot \rho_{10} + \alpha_{20} \cdot \rho_{20} \tag{7}$$

IV. EQUATION OF MOTION OF THE TUBE

Suppose there is a cylindrical directly axis tube of radius R and thickness h in the unperturbed state. Next we write the equation of motion of the tube, assuming that the wall material elastic orthotropic, fraction $h/R << 1$, and the tube is rigidly attached to the situation. Under these conditions it is adequate to use the following equation

$$p = \frac{hE_2}{(1-\nu_1\nu_2)R^2} w + \rho^* h \frac{\partial^2 w}{\partial t^2} \tag{8}$$

where, ρ^* is density of the wall, E_2 is the tangent Young's modulus, ν_1 and ν_2 are Poisson's ratios, where $E_1 \nu_2 = E_2 \nu_1 .(1-\nu_1\nu_2)^{-1}$ in the last equation is needed to account for bonds that prevent axial movement of the tube.

The second term in Equation (8) is the inertia of the tube. Its influence is generally considered to be insignificant. This follows to:

$$w = \frac{(1-\nu_1\nu_2)R^2}{hE_2} p \tag{9}$$

So, we can assume that Equations (1)-(3) and (9) represent a closed system of hydro elastic, which can be used to describe the evolution of small perturbations in the tube containing the gas and liquid media.

It is typical for the given situation to seasonably combine the derived system into an equation in relation to the desired function $\rho(x,t)$. For this purpose we proceed as follows: with the help of Equations (1) and (2) we eliminate the function $u(x,t)$ .As a result, we achieve

$$\frac{2}{R}\frac{\partial w}{\partial t} + \frac{\partial u}{\partial x} + \frac{1}{\rho_0}\frac{\partial \rho}{\partial t} = 0$$

Considering (9) and (3), after simple transformations, we finally write the original equation of the problem:

$$\left(1+\frac{a^2}{c_0^2}\right)\frac{\partial^2 \rho}{\partial t^2} + \frac{\xi}{\rho_0 c_0^2}\frac{\partial^3 \rho}{\partial t^3} - -a^2 \frac{\partial^2 \rho}{\partial x^2} - \frac{\xi}{\rho_0}\frac{\partial^3 \rho}{\partial x^2 \partial t} = 0 \tag{10}$$

where, $c_0^2 = \dfrac{hE_2}{2R\rho_0(1-\nu_1\nu_2)}$ is assigned to shorten the aforementioned equation.

Now the further course of analysis is to reduce the solution of partial differential (10) down to the solution of ordinary differential equation.

V. WAVE SOLUTION OF EQUATION

As we know, harmonic analysis is used to describe the complex impulses that are typical to wave motions, i.e., the impulses of complex structure are broken down to sinusoidal components, which form a Fourier series. Due to the linearity and uniformity of the defining equation, origination of each harmonic with frequency $n\omega$ is traced. Herein n is a natural number.

We can therefore conclude that consideration of the purely sinusoidal oscillation with a given frequency ω is crucially important. Therefore, using the method of separation of variables, we can find the solution of the Equation (9) in the following class of functions:

$$\rho(x,t) = y(x) \cdot \exp(i\omega t) \tag{11}$$

where, $y(x)$ is the unknown, in general, a complex function, and $i = \sqrt{-1}$ is the imaginary unit. Substituting (1) with (10), introducing the notation

$$\delta^2 = \frac{m_1 + i m_2}{a^2 + i m_3} \tag{12}$$

where

$$m_1 = \left(1+\frac{a^2}{c_0^2}\right)\omega^2, \quad m_2 = \frac{\xi\omega^3}{\rho_0 c_0^2}, \quad m_3 = \frac{\xi}{\rho_0}\omega$$

we derive

$$y''(x) + \delta^2 y(x) = 0 \tag{13}$$

where, y'' is the second derivative of the y on the coordinate x.

Separating the dispersion equation between real and imaginary parts, we write the following:

$$\delta^2 = \frac{m_1 + i\,m_2}{a^2 + i\,m_3} = \frac{m_1\,a^2 + m_2\,m_3}{a^4 + m_3^2} - \\ -i\,\frac{m_1\,m_3 - m_2\,a^2}{a^4 + m_3^2} = k_1 - i\,k_2 \tag{14}$$

According to the square root rule of a complex number, based on (14), we can determine the δ is $\delta = \pm(\delta_0 - i\,\delta_1)$, where, $\delta_0 = \sqrt{\frac{r + k_1}{2}}$, $\delta_1 = \sqrt{\frac{r - k_1}{2}}$ and $r = \sqrt{k_1^2 + k_2^2}$.

In further analysis, we will use the square root for which $\operatorname{Im}\delta < 0$. Therefore, we can set $\delta = \delta_0 - i\,\delta_1$, where δ_1 characterizes the damping of the wave along the tube. The general solution of Equation (14) is well known and is written as:

$$y = Ae^{-i\delta x} + Be^{i\delta x} \tag{15}$$

where, A, B are constants of integration (in general, complex), which are defined from the boundary conditions of the problem. It is enough that if we take waves which spread only positive direction of X axes as $y = Ae^{-i\delta x}$. Now obviously we have,

$$\rho(x,t) = A\exp[-i(\delta\,x - \omega t)] \tag{16}$$

Following Equations (3) and (9), we rewrite the relations for pressure and displacement. They turn into the form of:

$$p(x,t) = \left(a^2 + i\,m_3\right)\cdot A\exp\left[-i\left(\delta\,x - \omega t\right)\right] \\ w(x,t) = \frac{R}{2\,\rho_0\,c_0^2}\left(a^2 + i\,m_3\right)\cdot A\exp\left[-i\left(\delta\,x - \omega t\right)\right] \tag{17}$$

It is left to find an easy way to calculate the function $u(x,t)$. The easiest way is to set $u(x,t)$ as

$$u(x,t) = \upsilon(x)\cdot\exp(i\,\omega t) \tag{18}$$

Inserting this relation into the impulse Equation (1), the function $u(x,t)$ can easily be determined and thus, we can record:

$$u(x,t) = \frac{\delta}{\omega\,\rho_0}\left(a^2 + i\,m_3\right)\cdot A\exp[-i(\delta\,x - \omega t)] \tag{19}$$

Considering, flow rate we can write,

$$Q(x,t) = \pi\,R^2\,u(x,t) \tag{20}$$

From border condition we obtain,

$$Q(0,t) = Q^v\exp(i\,\omega t) \\ A = \frac{Q^v}{\pi\,R^2}\cdot\frac{\rho_0}{\delta\cdot\left(a^2 + i\,m_3\right)}\,\omega \tag{21}$$

Considering last equation in the Equations (16)-(19) we can obtain for velocity, density and displacement of mixture:

$$\rho(x,t) = \frac{Q^v}{\pi\,R^2}\cdot\frac{\rho_0}{\delta\cdot\left(a^2 + i\,m_3\right)}\,\omega\exp\left[-i\left(\delta\,x - \omega t\right)\right] \\ p(x,t) = \frac{Q^v}{\pi\,R^2}\cdot\frac{\rho_0}{\delta}\,\omega\exp\left[-i\left(\delta\,x - \omega t\right)\right] \\ w(x,t) = \frac{Q^v}{2\,\pi\,R\,\delta\,c_0^2}\cdot\omega\cdot\exp\left[-i\left(\delta\,x - \omega t\right)\right] \\ u(x,t) = \frac{Q^v}{\pi\,R^2}\cdot\omega\cdot\exp\left[-i\left(\delta\,x - \omega t\right)\right] \tag{22}$$

Hence, in accordance with the Euler formula for the amplitudes of the desired functions, we can write:

$$|\rho| = \frac{Q^v\cdot\rho_0\cdot\omega}{\pi\,R^2\,\sqrt{\left(\delta_1^2 + \delta_0^2\right)\cdot\left(a^4 + m_3^2\right)}}\cdot\exp(-\delta_1\,x) \\ |p| = \frac{Q^v\cdot\rho_0\cdot\omega}{\pi\,R^2\,\sqrt{\left(\delta_1^2 + \delta_0^2\right)}}\cdot\exp(-\delta_1\,x) \\ |w| = \frac{Q^v\cdot\rho_0\cdot\omega}{2\,\pi\,R\,c_0^2\,\sqrt{\left(\delta_1^2 + \delta_0^2\right)}}\cdot\exp(-\delta_1\,x) \\ |u| = \frac{Q^v\cdot\omega}{\pi\,R^2}\cdot\exp(-\delta_1\,x) \tag{23}$$

VI. THE OBTAINED NUMERICAL VALUES OF THE SOLUTION OF THE ISSUE.

For numerical calculations of the problem we define the parameters of the system corresponding to the experimental data for the rubber tube with parameters $E_2 = 4\cdot10^5\,\text{N/m}^2$, $\nu_1 = 0.3$, $\nu_2 = 0.1$, $h = 0.002\text{m}$, $R = 0.012\text{m}$, $\omega = 10^{-1}\,\text{sec}^{-1}$, $Q^v = 4\cdot10^{-6}\,\text{m}^3/\text{sec}$. Also, let's assume that $\mu = 0.11\cdot10^{-2}$ kg/m.sec, $p_0 = 10^5\,\text{N/m}^2$.

The tube is filled with mixture of water and small amount of additive of air bubbles. Considered four different fluids with differ densities such as carrier phases. For each mixture graph of dependence of amplitude on amount of number bubbles in unit volume is constructed. Evolution is taken for $0.01 \div 0.1$ value of α_{20}. Obtained amounts are compared at 0; 0.05; 0.1 value of X don't affect values of amplitude. Change of amplitude by the its dependence on amount of bubbles is enough as it's seen by graphs.

Figures 1, 2 and 3 show dependences of the density, displacement and hydrodynamic pressure, respectively related to the two-phase liquid on the size concentration of the bubbles, which are tested on 1- glycerin, 2- water, 3- ethanol and 4- oil.

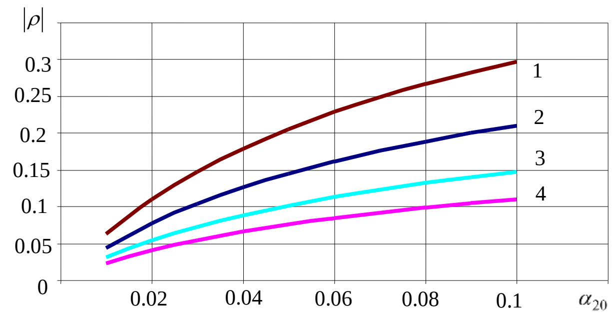

Figure 1. Dependence of the density of the two-phase liquid on the size concentration of the bubbles; 1- glycerin, 2- water, 3- ethanol, 4- oil

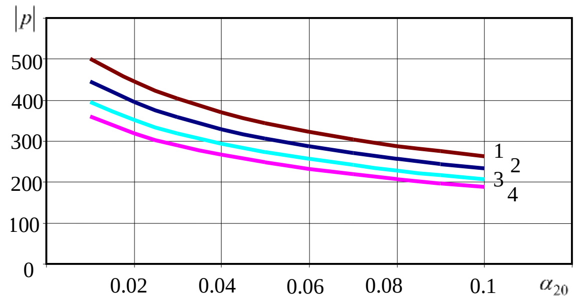

Figure 2. Dependence of the hydrodynamic pressure of the two-phase liquid on the size concentration of the bubbles; 1- glycerin, 2- water, 3- ethanol, 4- oil

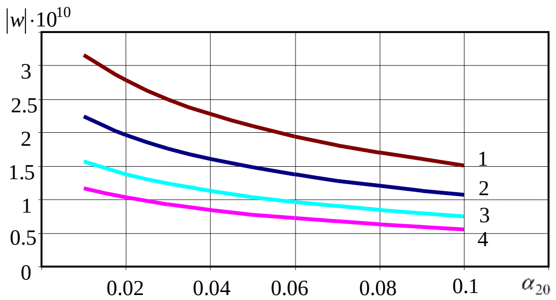

Figure 3. Dependence of the displacement of the two-phase liquid on the size concentration of the bubbles; 1- glycerin, 2- water, 3- ethanol, 4- oil

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