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# GEOMETRICAL NONLINEAR VIBRATION OF FUNCTIONALLY-GRADED LONGITUDINALLY STRENGTHENED, FLOWING FLUID-CONTACTING CYLINDRICAL SHELL

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**Abstract-** In the present paper we study geometrical nonlinear vibrations of a moving fluid-contacting functionally-graded cylindrical shell. Using the Hamilton-Ostrogradsky vibrational principle, the finding of vibration frequencies of the considered system is reduced to the solution of the system of differential equations and is realized by the numerical method.

**Keywords:** Functionally-Graded Material, Cylindrical Shell, Geometrical Nonlinear Vibration, Ostrogradsky-Hamilton Action.

## I. INTRODUCTION

Functionally-graded materials are widely used in manufacturing numerous objects, in particular aerocosmic objects. The objects made of these materials re used in contact with high temperature media. Therefore, application of these constructions with regard to liquid medium is of great importance. Sometimes, it is necessary to strengthen the mentioned constructions. The problem of vibrations and stability of shells made of functionally-graded materials ignoring the influence of liquid medium found their solutions [1-6]. The reference [7] deals with geometrical nonlinear vibrations of a rectangular plate made of functionally-graded material. The reference [10] was devoted to study of linear vibrations of a functionally-gradient shell.

## II. PROBLEM STATEMENT

Let us consider a fluid-contacting cylindrical shell made of mixture of ceramics and metal. As in the reference [10] it is assumed that the fraction of ceramic material in the total volume changes by the law:

$$V=\left(\frac{2z+h}{2h}\right)^{k} \tag{1}$$

where, $h$ is the shell's thickness, $k$ is the power index of the fraction of the ceramic material in the volume and $0\le k\le\infty$. If $k=0$, the structure of the shell consists only of ceramics, if $k=\infty$, will consist of metal. We will assume that mechanical characteristics of materials (the Young modulus, density, etc.) change by the following law [4, 5]:

$$P_j=P_0\left(P_{-1}T^{-1}+1+P_1T+P_2T^2+P_3T^3\right)$$

where, the coefficients $P_0,P_{-1},P_1,P_2,P_3$ are specifically defined for each material. The values of these coefficients for some materials were given in the papers [1, 4, 5]. The mechanical properties of the mixture consisting of two parts are determined by the following formula:

$$P(z,T)=\begin{pmatrix}P_c(T)-\\ P_m(T)\end{pmatrix}\left(\frac{z}{h}+\frac{1}{2}\right)^{k}+P_m(T) \tag{2}$$

By Equation (2) we can calculate the elasticity modulus $E$ of the mixture, the Poisson ratio $\nu$ and density $\rho$, where, $P_c(T)$ and $P_m(T)$ are the characteristics of ceramics and metal.

The system of motion equations of medium-contacting composite cylindrical shell is found from the stationarity condition of Hamilton-Ostrogradsky action:

$$\delta W=0 \tag{3}$$

where, $W=\int_{t'}^{t''}Ldt$ is Hamilton's action, $L=K-\Pi$ is the Lagrange function, $t'$ and $t''$ are the given any moments of time. The potential and kinetic energies of the system are as follows:

$$\begin{gathered}U=\frac{1}{2}\iint_{\Omega}\begin{pmatrix}N_{11}\varepsilon_{11}+N_{22}\varepsilon_{22}+N_{12}\varepsilon_{12}+\\ M_{11}\chi_{11}+M_{22}\chi_{22}+M_{12}\chi_{12}\end{pmatrix}d\Omega+\\ +\frac{1}{2}\iint_{\Omega}\left(\varrho_x\left(w,_x+\psi_x\right)+\varrho_y\left(w,_y+\psi_y\right)\right)d\Omega\\ T=\frac{1}{2}\iint_{\Omega}[I_0\left(u_t^2+v_t^2+w_t^2\right)+2I_1\begin{pmatrix}u_t\psi_{x,t}+\\ v_t\psi_{y,t}\end{pmatrix}+\\ +I_2\left(\psi_{x,t}^2+\psi_{y,t}^2\right)]dxdy\end{gathered} \tag{4}$$

where,

$$I_0=\left(\rho_m+\frac{\rho_c-\rho_m}{k+1}\right)h$$

$$I_1=\int_{-h/2}^{h/2}\rho(z)zdz=\frac{(\rho_c-\rho_m)k}{2(k+1)(k+2)}h^2$$

$$I_2 = \int_{-h/2}^{h/2} \rho(z) z^2 dz = \left( \begin{array}{c} \frac{\rho_m}{12} + (\rho_c - \rho_m) \\ \left( \frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+4)} \right) \end{array} \right) h^3$$

$$A_0 = -\iint_{\Omega} pwdxdy$$

In Equation (4), we have:

$\varepsilon_{ij} = \varepsilon_{ij}^L + \varepsilon_{ij}^{ND} \quad (i\ , j = 1,2)$,

$\varepsilon_{11}^L = u_x + w/R_x$ , $\varepsilon_{22}^L = v_y + w/R_y$ , $\varepsilon_{12}^L = u_y + v_x$ ,

$\varepsilon_{11}^{ND} = \frac{1}{2} w_x^2$ , $\varepsilon_{22}^{ND} = \frac{1}{2} w_y^2$ , $\varepsilon_{12}^{ND} = w_x + w_y$ ,

$\varepsilon_{13} = w,_x + \psi_x$ , $\varepsilon_{23} = w_y + \psi_y$ , $\chi_{11} = \psi_{x,x}$ ,

$\chi_{22} = \psi_{y,y}$ $\chi_{12} = \psi_{x,y} + \psi_{y,x}$ ,

$$N = \{N_{11}; N_{22}; N_{12}\}^T = \frac{1}{1-\nu^2} [C](E_1 \varepsilon + E_2 \chi) ,$$

$$M = \{M_{11}; M_{22}; M_{12}\}^T = \frac{1}{1-\nu^2} [C](E_2 \varepsilon + E_3 \chi) ,$$

$$[C] = \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} , \; E_1 = \left( E_m + \frac{E_c - E_m}{k+1} \right) h ,$$

$$E_2 = \frac{(E_c - E_m) kh^2}{2(k+1)(k+2)} ,$$

$$E_3 = \left( \frac{E_m}{12} + (E_c - E_m) \left( \begin{array}{c} \frac{1}{k+3} - \frac{1}{k+2} + \\ \frac{1}{4(k+4)} \end{array} \right) \right) h^3 ,$$

$$\rho = \left( \rho_m + \frac{\rho_c - \rho_m}{k+1} \right) h .$$

Cutting forces $Q_x$ and $Q_y$ are determined from the expressions $Q_x = K_S^2 A_{33} \varepsilon_{13}$ and $Q_y = K_S^2 A_{33} \varepsilon_{23}$. The coefficient $K_S^2$ is called the regularizing coefficient. In the calculation process we accept $K_S^2 = \frac{5}{6}$. $A_0$ is the inverse sign work done by pressure force $p$ on the shell in displacement $w$ of the shell. Pressure force $p$ is determined from the motion equation of ideal fluid moving with velocity $U$ :

$$\Delta \tilde{\phi} - \frac{1}{a_0^2} \left( \frac{\partial^2 \tilde{\phi}}{\partial t^2} + 2U \frac{\partial^2 \tilde{\phi}}{R \partial \xi \partial t} + U^2 \frac{\partial^2 \tilde{\phi}}{R^2 \partial \xi^2} \right) = 0 \qquad (5)$$

In shell-fluid contact, in radial direction the equality of velocity and pressure is satisfied:

$$\vartheta_r \big|_{r=R} = \frac{\partial \tilde{\phi}}{\partial r} \bigg|_{r=R} = -\left( \omega_0 \frac{\partial w}{\partial t^1} + U \frac{\partial w}{R \partial \xi} \right) \qquad (6)$$

$$q_z = -p\big|_{r=R} \qquad (7)$$

Look for the $\tilde{\phi}$ as potential of perturbations in form:

$$\tilde{\phi}(\xi, r, \theta, t_1) = f(r) \cos n\theta \sin kx \sin \omega t \qquad (8)$$

where, $n$, $k$ are wave numbers in the direction of coordinate axes, $\omega$ is an unknown frequency, $f(r)$ is an unknown function. Using Equations (6), (7) and (8):

$$\begin{aligned} \tilde{\phi} &= -\Phi_{\alpha n} \left( w_0 \frac{\partial w}{\partial t^1} + U \frac{\partial w}{R \partial \xi} \right) \\ p &= \Phi_{\alpha n} \rho_m \left( \omega_0^2 \frac{\partial^2 w}{\partial t_1^2} + 2U \frac{\partial^2 w}{R \partial \xi \partial t_1} + U^2 \frac{\partial^2 w}{R^2 \partial \xi^2} \right) \end{aligned} \qquad (9)$$

where,

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r) / I_n(\beta r) & , M_1 < 1 \\ J_n(\beta_1 r) / J_n(\beta_1 r) & , M_1 > 1 \\ R^m / nr^{n-1} & , M = 1 \end{cases} \qquad (10)$$

In Equation (10) $M_1 = \frac{U + \omega / m}{a_0}$, $\beta^2 = R^{-2} \left(1 - M_1^2\right) \chi^2$, $\beta_1^2 = R^{-2} \left(M_1^2 - 1\right) \chi^2$, $J_n$ is the first kind $n$th order Bessel function, $J_n'$ is its derivative with respect to variable $r$, $I_n$ is the $n$th order modified Bessel function, $I_n'$ is its derivative with respect to variable $r$, $a_0$ is the rate of sound propagation in fluid.

Let us write total energy of a construction consisting of longitudinal ribs made of homogeneous metal [12]:

$$\begin{aligned} J &= \frac{1}{2} \sum_{i=1}^{k_1} \int_{x_1}^{x_2} \left[ \begin{array}{l} \tilde{E}_i F_i \left( \frac{\partial u_i}{\partial x} \right)^2 + \tilde{E}_i J_{zi} \left( \frac{\partial^2 \vartheta_i}{\partial x^2} \right)^2 + \\ \tilde{E}_i J_{yi} \left( \frac{\partial^2 w_i}{\partial x^2} \right)^2 + \tilde{G}_i J_{kpi} \left( \frac{\partial \phi_{kpi}}{\partial x} \right)^2 \end{array} \right] dx + \\ &+ \sum_{i=1}^{k_1} \rho_i F_i \int_{x_1}^{x_2} \left[ \begin{array}{l} \left( \frac{\partial u_i}{\partial t} \right)^2 + \left( \frac{\partial \vartheta_i}{\partial t} \right)^2 + \\ \left( \frac{\partial w_i}{\partial t} \right)^2 + \frac{J_{kpi}}{F_i} \left( \frac{\partial \phi_{kpi}}{\partial t} \right)^2 \end{array} \right] dx \end{aligned} \qquad (11)$$

where, $\tilde{E}_i$ is modulus of elasticity of longitudinal ribs; $F_i$ are the square of cross sections of longitudinal ribs; $J_{yi}, J_{kpi}$ are the inertia moments of longitudinal ribs; $\tilde{G}_i$ shear elasticity modules of longitudinal ribs; $u_i, \vartheta_i, w_i$ are displacements of the points of longitudinal ribs; $k_i$ is the amount of longitudinal ribs.

It is assumed that the rigid contact conditions between the shell and ribs are satisfied

$$\begin{aligned} & u_i(x) = u(x, y_i) + h_i \phi_1(x, y_i) \\ & \vartheta_i(x) = \vartheta(x, y_i) + h_i \phi_2(x, y_i) \\ & w_i(x) = w(x, y_i), \phi_i(x) = \phi_1(x, y_i) \\ & \phi_{kpi}(x) = \phi_2(x, y_i);\; h_i = 0{,}5h + H_i^1 \end{aligned} \qquad (12)$$

where, $H_i^1$ is the distance of the axis of the *i*th bar from the surface of cylindrical shell, $\phi_i$, $\phi_{kpi}$ are turing and rotational angles of cross sections of the bar and are defined by the displacements of the shell as follows:

$$\phi_{kpi}(x) = \phi_2(x, y_i) = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R}\right)\Bigg|_{y=y_i}$$

We will accept that the cylindrical shell was highly supported, i.e. in the sections *x*=0 and *x*=*L* the following conditions are satisfied:

$u = 0$ , $w = 0$ , $T_1 = 0$ , $M_1 = 0$

where, $T_1$, $M_1$ is the force and moment acting on cross section of the cylindrical shell.

Using the stationarity condition of Ostrogodsky-Hamilton action, if we realize the variation process in the equality $\delta W = 0$ and take into account arbitrariness and independence of variations $\delta u$, $\delta v$, $\delta w$, we get a frequency equation of a cylindrical shell dynamically contacting with fluid.

Thus, the solution of problem of geometrical nonlinear vibrations of a cylindrical shell dynamically contacting with fluid is reduced to integration of total energy of a construction consisting of a cylindrical shell with fluid-filled inner domain.

### III. PROBLEM SOLUTION

We look for displacements of the shell as follows:

$$u = u_0(t)\cos\chi\xi\cos n\theta$$
$$\vartheta = \vartheta_0(t)\sin\chi\xi\sin n\theta \qquad (13)$$
$$w = w_0(t)\sin\chi\xi\cos n\theta$$

where, $u_0, \vartheta_0, w_0$ are unknown functions, $\chi, n$ are generator and wave number of the cylindrical shell in peripheral direction, $\xi = x/L$ .

Allowing for (13), (11), (4) substituting Equation (12) in (3), from the stationarity condition of Hamilton-Ostrogradsky action, with respect to the unknown functions $u_0, \vartheta_0, w_0$ we get the system of second order differential equations. As the system is of bulky form, we don't cite it here. If we look for the solution of this system in a first approximation in the form $u_0 = u_1\sin\omega t$, $\vartheta_0 = \vartheta_1\sin\omega t$, $w_0 = w_1\sin\omega t$, we find the dependence between the sought-for frequency $\omega$ and $u_1, \vartheta_1, w_1$. By means of this dependence we can construct the skeleton curve. In calculations for the parameters the following estimations [7] were taken:

$E_m / E_c = 70380$ ; $\nu_m = \nu_c = 0.3$ ; $\rho_m / \rho_c = 2707/3800$ ; $\rho_0 / \rho = 0.115$ ; $a_0 = 1350$ m/san ; $E_i = 6.67\times10^9$ N/m$^2$ ; $\nu = 0.3$ ; $\chi = 1$ ; $n = 8$ ; $h_i = 1.39$ mm ; $R = 160$ mm ;

$\frac{F_i}{2\pi Rh} = 0.1591\times10^{-1}$ ; $\frac{I_{yi}}{2\pi R^3 h} = 0.8289\times10^{-6}$ ; $h = 0.45$ mm ;

$\frac{I_{kpi}}{2\pi R^3 h} = 0.5305\times10^{-6}$ ; $J_{zi} = 5.1$ mm$^4$ ; $L = 800$ mm .

The dependence of the ratio of frequency of nonlinear vibrations on the curvature of the shell is depicted in Figure 1. The dependence of linear frequency parameter $\Omega_x = \omega h\sqrt{\rho_c / E_c}$ on the power index $k$ of fraction of the ceramic material in the volume was given in Figure 2. As is seen from figure 1, as the shell's curvature increases, the frequencies of nonlinear vibrations also increase. As the power index $k$ of the fraction of ceramic material in the volume increases, as is seen from Figure 2, the frequency of linear vibrations decreases.

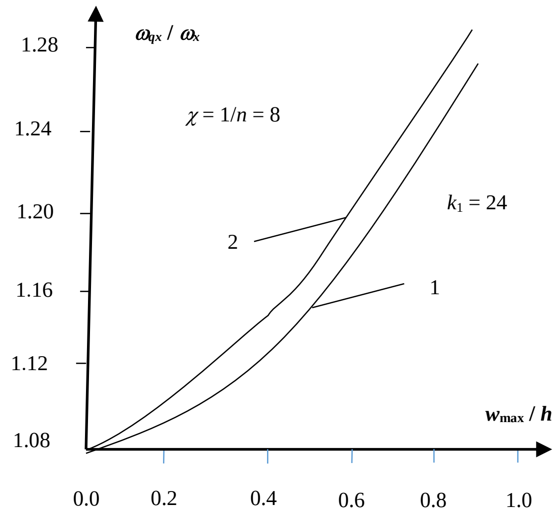


Figure 1. Dependence of frequencies of nonlinear vibrations of the system on shell's curvature

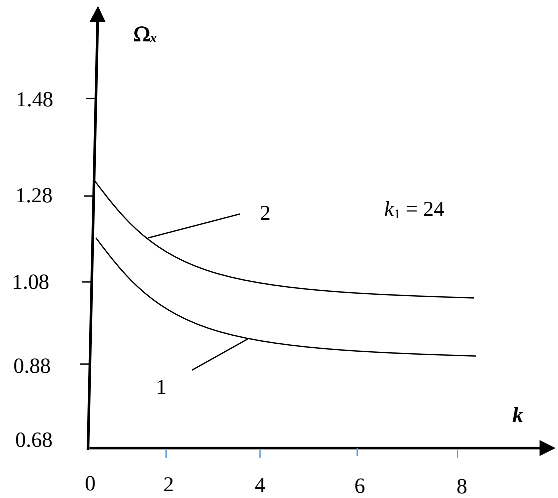


Figure 2. Dependence of frequencies of linear vibrations of the system on power index

In Figures 1 and 2, curve 1 corresponds to account of influence of fluid on vibration process, curve 2 to no fluid cases. As is seen, account of the influence of fluid reduces to decrease of frequencies of natural vibrations of the system. Calculations show that natural vibrations of the system increase according to the increase of the number of longitudinal ribs.

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