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ULTIMATE LOAD OF THREE-LAYERED ANNULAR PLATE REINFORCED WITH SYSTEMS OF FIBERS

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Abstract- A problem of determination of ultimate load of an annular three-layered plate whose middle layer was reinforced with four systems of fibers is studied in this paper. The inner contour of the plate is simply supported, the external one is built-in. It is shown that the plate is divided into five annular zones, and in each of these zones different plastic states are realized. Static fields of moments are determined, the equations for unknown radii between plastic zones and also equations for determining reactions at supports and ultimate loads are determined.

Keywords: Three-Layered Composite, Bending, Load-Bearing Capacity, Simply-Supported, Built-in Contour.

I. INTRODUCTION

The constructions made of composite materials are widely used in many branches of national economy. This is an artificially created homogeneous material consisting of multiple layers, reinforcing fibers and fillers. The filler fastens the fibbers and together with layers gives to product a stable spatial form.

The fibers take on themselves the most part of the load, the layers take a part of loads and other impacts. At present, more than one third of all aircrafts are manufactured from composite materials. When producing aircrafts, epoxy resin is most used as a filler, a glass fiber and carbon fiber are used as reinforcing fibers. Composite materials have significant advantages over metals, wood and textile.

These are light weight, the possibility of creating very smooth and complex curved, well-streamlined aerodynamic surface, absence of corrosion, low level of fatigue during long-term deformations, concealing radar signatures, etc. [1]. Application of glass fiber-based composite materials in Airbus A380 led to reduction of aircraft's weight by 15 tons compared with aluminum with preservation or improvement of strength properties. Approximately one million finest fibers cross from one cm^2 cross section of the composite propeller of the newest USA helicopters propellers. Being excellent from the point of view of structural rigidity, the reinforced composites are weak in penetration of moisture, aggressive media and also exposure to high temperature. All these, makes actual the study of load-bearing capacity of multi-layered fibrous composite materials.

In [2], hypersurfaces of fluidity of a three-layered composite shell whose middle surface was reinforced with fibers, the external layers defend the construction from negative impact and simultaneously strengthen it, were built. Load-bearing capacity of a three-layered fibrous annular composite plate at various conditions of built-in along the contour, was studied in [3-8]. Statically admissible fields of bending moments and kinematic admissible fields of deflection rate were determined.

II. PROBLEM STATEMENT

The load-bearing capacity of an annular three-layered composite plate simply supported along the inner contour and built-in along the external contour is studied in the paper. The middle layer of the composite is reinforced with four systems of thin fibers arranged in the form of films at various distances from the middle of the layer. It is assumed that plate is subjected to annular concentrated external load in the lateral direction.

Equilibrium equation in dimensionless variables has the form [3-8]

$$(rm_1)' - m_2 = -T^{ar} + Ta \quad \left(T^{ar} = \int_a^r p(\eta)\eta \, d\eta \right) \tag{1}$$

$a < r < b$

where the prime indicates the derivative with respect to r, m_1 and m_2 are dimensionless principal bending moments at radial and peripheral direction, $p = p(r)$ is the dimensionless external load, T is the unknown linear load in the internal contour of the plate.

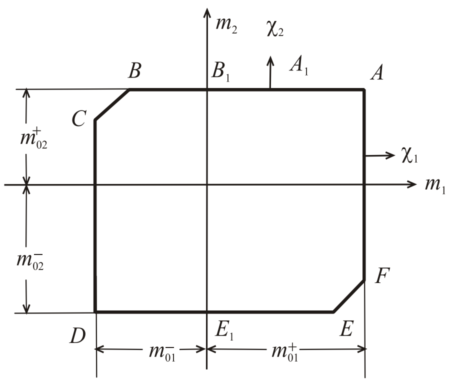

Figure 1. Fluidity hexagon

Equation (1) must be solved under the following boundary conditions: on simply supported edge $r=a$ $m_1=0$, $w=0$; at the built-in edge $r=b$ $w=0$, $dw/dr=0$ or $m_1=-m_{10}^{-}$, were $w(r)$ is deflection rate.

Equation (1) is an ordinary differential equation with two unknowns m_1 and m_2. The second equation between these quantities is given by the plastic yield condition that was constructed in [2] (Figure 1). For the sides AB and AF of the hexagon we have the following limit values of positive and negative bending moments

$$m_i=m_{0i}^{+}=c_0+c_{1i}^{+}s_{0i}-c_{2i}s_{0i}^2+c_3q_0-c_4q_0^2-c_{5i}s_{0i}q_0 \quad (2)$$

for the sides CD and DE

$$m_i=-m_{0i}^{-}=-[c_0+c_{1i}^{-}s_{0i}-c_{2i}s_{0i}^2+c_3q_0-c_4q_0^2-c_{5i}s_{0i}q_0] \quad (3)$$

and for the sides EF and BC

$$m_2=\alpha m_1+\beta_1 \ , \quad m_2=\alpha m_1+\beta_2 \quad (4)$$

respectively. Here for the coefficients we used the following denotation:

$$c_0=\frac{2k}{1+k}\ ,\ c_{2i}=\frac{2(1-\mu_i)^2}{1+k}\ ,\ c_3=4\frac{1+vk}{1+k}\ ,\ c_4=\frac{2(1-v)^2}{1+k}$$

$$c_{5i}=\frac{4(1-\mu_i)(1-v)}{1+k}\ ,\ i=1,2$$

$$c_{1i}^{-}=4\left(d_i'+\mu_i d_i''+\frac{1-\mu_i}{2}\frac{1-k}{1+k}\right)\ ,\ i=1,2$$

$$\alpha=\frac{(1-k)[(1-\mu_2)s_{02}+(1-v)q_0]+k}{(1-k)[(1-\mu 1)s_{01}+(1-v)q_0]+k}$$

$$q_0=\frac{Q_0}{\sigma_0H^2}\ ,\ \beta_1=a_2-\alpha a_1\ ,\ \beta_2=a_4-\alpha a_3\ ,\ s_{0i}=\frac{S_{0i}}{\sigma_0H^2}$$

$$a_1=\frac{1}{1-k^2}\left\{k(1-k)+(1+k^2)[(1-\mu_1)s_{01}+(1-v)q_0]-\right.$$
$$\left.-2k[(1-\mu_2)s_{02}+(1-v)q_0\right\}+4(d_1''+\mu_1d_1')s_{01}^{+}+2(1+v)q_0$$

$$a_2=-\frac{1}{1-k^2}\left\{k(1-k)-(1+k^2)[(1-\mu_2)s_{02}+(1-v)q_0]+\right.$$
$$\left.2k[(1-\mu_1)s_{01}+(1-v)q_0\right\}-4(d_2'+\mu_2d_2'')s_{02}-2(1+v)q_0$$

$$a_3=-a_1-4(1-\mu_1)(d_1'-d_1'')s_{01}$$

$$a_4=-a_2-4(1-\mu_2)(d_2'-d_2'')s_{02}$$

where, d_i' and d_i'' are dimensionless distances (referred to the thickness H) from the middle surface to the upper and lower layers of the fibers, k and v are the ratios of ultimate loads of the matrix and external layers at extension and compression, respectively, s_{0i}^{+} and s_{0i}^{-} are dimensionless forces for fibers at extension and compression, respectively, q_0 is dimensionless yield point of material of external layers at compression [2].

III. PROBLEM SOLUTION

For the given type of load q (directed down) from boundary conditions it follows that radial bending moment will have positive value (extension of lower and compression of upper layers) right up to the area adjacent to the built-in external contour, where it changes sign. In this case, plastic state of the plate is determined by the side E_1E of the fluidity hexagon near the inner edge $r=a$, on which $m_1=0$ and $m_2=-m_{20}^{-}$, while on the contour $r=b$ $m_1=-m_{01}^{-}$. Thus, we must to look for the solution of the problem according to the following sequence of fluidity regimes E_1E-EF-FA-AB-BC. Then the plate is divided into five annular zones in which the yield condition is linear and Equation (1) is easily integrated.

Assume that the load $p(r)$ concentrated on a circle of radius $c\in(a,b)$ is given in the form $p(r)=p\delta(r-c)$, where $\delta(x)$ is Dirac's delta function [6]. Calculating the integral

$$T^{ar}=p\int_a^r\delta(\eta-c)\eta\,d\eta=pcI(r-c)$$

where $I(r-c)$ is Heaviside's unit function, we rewrite Equation (1) in the form

$$(rm_1)'-m_2=-pcI(r-c)+Ta \quad (5)$$

On the section $a\le r\le\rho_1$ the plastic regime E_1E is realized, according to which $m_2=-m_{20}^{-}$. Substituting this into Equation (5), after integration we have

$$r\,m_1=(-m_{20}^{-}+Ta)r-pc(r-c)I(r-c)+C$$

After determining the constant C from the condition $m_1(a)=0$, we find

$$r\,m_1=(-m_{20}^{-}+Ta)(r-a)-pc(r-c)I(r-c) \quad (6)$$

From (6) it is seen that provided $a<c<\rho_1$, on the circumference $r=c$ the radial moment does not go a jump, and its derivative dm_1/dr possesses the jump $\frac{dm_1}{dr}=-p$, that is consistent with the equilibrium Equation (5). Here the square bracket means a jump, i.e. the difference of values of the corresponding quantity in the right and left hand side of the point under consideration. Determining $m_1(\rho_1)$ from (6) and substituting in formula $m_2=\alpha m_1+\beta_1$, as a result we get $-m_{20}^{-}$, which will result in

$$m_{20}^{-}\left(\alpha-1-\alpha\frac{a}{\rho_1}\right)=b_1+Ta\alpha\frac{\rho_1-a}{\rho_1}-$$
$$-\frac{\alpha}{\rho_1}pc(\rho_1-c)I(\rho_1-c) \quad (7)$$

In the domain $\rho_1\le r\le\rho_2$ we have the state EF, at which $m_2=\alpha m_1+\beta_1$. Equation of equilibrium (5) takes the form $rm_1'+(1-\alpha)m_1=(Ta+\beta_1)-pcI(r-c)$.

The solution of this equation i

$$m_1=Cr^{\alpha-1}+\frac{Ta+\beta_1}{1-\alpha}-\frac{pc}{1-\alpha}\left[1-\left(\frac{c}{r}\right)^{1-\alpha}\right]I(r-c)$$

Determining an arbitrary constant C from continuity condition $m_1(\rho_2)=m_{10}^{+}$, we get

$$m_1(r)=m_{10}^+\left(\frac{r}{\rho_2}\right)^{\alpha-1}+\frac{Ta+\beta_1}{1-\alpha}\left[1-\left(\frac{r}{\rho_2}\right)^{\alpha-1}\right]+ \\ +\frac{pc}{1-\alpha}\left\{\left[\left(\frac{r}{c}\right)^{\alpha-1}-1\right]I(r-c)+\left[1-\left(\frac{c}{\rho_2}\right)^{1-\alpha}I(\rho_2-c)\right]\right\} \quad (8)$$

It is easy to see that even in this domain under the condition $\rho_1<c<\rho_2$ on the circumference $r=c$ radial moment does not undergo a jump, while its derivative dm_1/dr has a jump equal to $-p$. Using continuity condition $m_1(\rho_1)=0$, we get

$$(-m_{20}^-+Ta)\left(1-\frac{a}{\rho_1}\right)-pc\left(1-\frac{c}{\rho_1}\right)I(\rho_1-c)= \\ =m_{10}^+\left(\frac{\rho_1}{\rho_2}\right)^{\alpha-1}+\frac{Ta+\beta_1}{1-\alpha}\left[1-\left(\frac{\rho_1}{\rho_2}\right)^{\alpha-1}\right]+\frac{pc}{1-\alpha}\times \\ \times\left\{\left[\left(\frac{\rho_1}{c}\right)^{\alpha-1}-1\right]I(\rho_1-c)+\left[1-\left(\frac{c}{\rho_2}\right)^{1-\alpha}\right]I(\rho_2-c)\right\} \quad (9)$$

When the stress state of plate corresponds to the side FA $(\rho_2\le r\le\rho_3)$, for deformation rates we have [1]

$$\chi_1=-w''\ge 0\ ,\ \chi_2=-\frac{1}{r}w'=0$$

The solution of these equations will be $w=w_0=$ const, i.e. the plate's annular part, $\rho_2\le r\le\rho_3$ moves in this domain as an absolute rigid body. Circumferences $r=\rho_2$ and $r=\rho_3$ are hinged circles, where the first derivate of deflection rate undergoes a rupture, deflection rate is continuous, while radial bending moment has a maximum value.

On the interval $\rho_3\le r\le\rho_4$ we have the state AB, at which $m_2=m_{20}^+$. From Equation (5) we get

$$r\,m_1=(m_{20}^++Ta)r-pc(r-c)I(r-c)+C$$

Here we determine the arbitrary constant C from condition $m_1(\rho_3)=m_{10}^+$, then

$$r\,m_1=(m_{20}^++Ta)(r-\rho_3)+m_{10}^+\rho_3+ \\ +pc\left[(\rho_3-c)I(\rho_3-c)-(r-c)I(r-c)\right] \quad (10)$$

Determining $m_1(\rho_4)$ from formula (10) and substituting in $m_2=\alpha m_1+\beta_2$, according to continuity condition we must get m_{20}^+; then

$$m_{20}^+\left(1-\alpha+\frac{\alpha\rho_3}{\rho_4}\right)=\beta_2+Ta\alpha\frac{\rho_4-\rho_3}{\rho_4}+\frac{\alpha\rho_3}{\rho_4}m_{10}^++ \\ +\frac{\alpha\,pc}{\rho_4}\left[(\rho_3-c)I(\rho_3-c)-(\rho_4-c)I(\rho_4-c)\right] \quad (11)$$

In the domain $\rho_4\le r\le b$ we have the state BC, at which $m_2=\alpha m_1+\beta_2$. Here the solution might be obtained from (8) by replacing the quantities m_{10}^+,ρ_2,β_1 by $-m_{10}^-,b,\beta_2$, respectively:

$$m_1(r)=-m_{10}^-\left(\frac{r}{b}\right)^{\alpha-1}+\frac{Ta+\beta_2}{1-\alpha}\left[1-\left(\frac{r}{b}\right)^{\alpha-1}\right]+ \\ +\frac{pc}{1-\alpha}\left\{\left[\left(\frac{r}{c}\right)^{\alpha-1}-1\right]I(r-c)+1-\left(\frac{c}{b}\right)^{1-\alpha}\right\} \quad (12)$$

Using the solutions (10) and (12) it is easy to show that if load circumference $r=c$ is in relevant domain, then on this circumference radial moment is continuous, while its derivative along the radial coordinate has a jump equal to $-p$. Taking into account $\alpha m_1(\rho_4)+\beta_2=m_{20}^+$ from Equation (8) for $r=\rho_4$ we find

$$\frac{m_{20}^+-\beta_2}{\alpha}=-m_{10}^-\left(\frac{\rho_4}{b}\right)^{\alpha-1}+\frac{Ta+\beta_2}{1-\alpha}\left[1-\left(\frac{\rho_4}{b}\right)^{\alpha-1}\right]+ \\ +\frac{pc}{1-\alpha}\left\{\left[\left(\frac{\rho_4}{c}\right)^{\alpha-1}-1\right]I(\rho_4-c)+1-\left(\frac{c}{b}\right)^{1-\alpha}\right\} \quad (13)$$

Now let's study the possibility of continuation of the static field on the annular domain $\rho_2\le r\le\rho_3$. Accepting that tangential moment m_2 and shear force are continuous functions, from the equilibrium equation we get that if circumference $r=c$ doesn't coincide with circumferences $r=\rho_2$ and $r=\rho_3$, then derivative dm_1/dr can't have a jump on them, i.e.

$$\frac{dm_1}{dr}=0 \quad \text{for } r=\rho_2 \text{ and } r=\rho_3,\ c\ne\rho_2,\rho_3 \quad (14)$$

as $m_1=m_{10}^+$ on these radii. But when we admit the possibility of step-wise change of the moment m_2, from the equilibrium Equation (1) we get

$$r\left[\frac{dm_1}{dr}\right]=[m_2]\ ,\quad r\ne c \quad (15)$$

As $[m_2]$ at $r=\rho_2$ and $r=\rho_3$ has positive values, while $[dm_1/dr]$ on these radii at $r\ne c$ may be only negative, then, fulfillment of the condition (15) is impossible. From this we conclude that moments field m_2 must be continuous in the domain of the plate $r\ne c$, i.e. $[m_2]=[dm_1/dr]=0$ at $r\ne c$.

Assuming $\rho_2<c<\rho_3$ and fulfilling the condition (14) with using derivative of Equation (8) at $r=\rho_2$ and derivative of formula (10) at $r=\rho_3$, we get

$$Ta=m_{10}^+(1-\alpha)-\beta_1 \quad (16)$$

$$m_{20}^+-m_{10}^+-pc+Ta=0 \quad (17)$$

Equation (16) determines the unknown reaction Ta, while (17) determines ultimate load, which we easily find

$$\frac{pc}{m_{20}^+}=1-\frac{\alpha m_{10}^++\beta_1}{m_{20}^+} \quad (18)$$

As seen, due to reinforced fibers and covers, the ultimate load of the plate increases by $1-\frac{\alpha m_{10}^++\beta_1}{m_{20}^+}$ times, because expression $\alpha m_{10}^{+} + \beta_1$ is negative, being the ordinate of the point F of fluidity hexagon in the plane $m_1 m_2$, $F(m_{01}^{+}, am_{01}^{+} + \beta_1)$. In the case $\mu_1 = \mu_2 = 1$ it is not difficult to get

$$\alpha m_{01}^{+} + \beta_1 = -\left[2q_0 \frac{1+\gamma k}{1+k} + 4\left(d_2' + d_2''\right) s_{02} \right]$$

As seen, ultimate load increases with the expression inside the square bracket. At $\gamma = 1$, $d_2' = d_2'' = \frac{1}{2}$ this expression has the greatest value $2q_0 + 4s_{02}$.

Equations (7), (9), (11) and (13) allow to determine unknown radii ρ_1, ρ_2, ρ_3 and ρ_4 . In the case $\rho_2 < c < \rho_3$, these equations are substantially simplified.

IV. CONCLUSION

The load bearing ability of an annular three-layered plate, whose central layer is reinforced with four systems of fibers is determined. All constituents of the plate have ideal plastic properties with different ultimate tensile and compressive forces. It is assumed that the plate is simply supported along the inner, built-in along the outer contour and is under the action of a concentrated annular load in the upper surface. Statically admissible fields of bending moments, the equations that determine ultimate load, support reaction and radii of domains corresponding to different plastic regimes, are determined.

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BIOGRAPHY

Akif Ali Jahangirov was born in 1963 in Baku, Azerbaijan. He graduated from "Motor-Transport" Faculty of Azerbaijan Polytechnic Institute (Baku, Azerbaijan) in 1986. He worked as a Chief Engineer at the Ministry of Motor-Transport (Baku, Azerbaijan) since 1986. He started to work as a Senior Laboratory Assistant at Department of "Organization of Road Haulage and Traffic" of Azerbaijan Technical University (Baku, Azerbaijan) in 1990. He received the Ph.D. degree at technical sciences in 1992. He was promoted to the rank of Associated Professor of the mentioned department in 1999. He was appointed as a Deputy Dean of "Motor-Mechanics" Faculty and also Deputy of the Rector of the same university in 2002. He is the Chairman of Trade Union Committee of the same university since 2006.