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OSCILLATIONS OF LONGITUDINALLY REINFORCED HETEROGENEOUS ORTHOTROPIC CYLINDRICAL SHELL WITH FLOWING LIQUID

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Abstract- The proposed article examines free oscillation of the longitudinally reinforced, orthotropic, and heterogeneous in thickness cylindrical shell being in contact with the flowing liquid. Using the Hamilton - Ostrogradsky's Variation Principle, the system of equations of motion is based on the longitudinal reinforcement, orthotropic, heterogeneous cylindrical shell being in contact with the flowing liquid. The heterogeneity of the shell material on thickness is taken into account, assuming that the Young's modulus and the density of the shell material are functions of the normal coordinates. In the study of free oscillation of a longitudinally reinforced, orthotropic, heterogeneous cylindrical shell contact with flowing liquid, two cases are considered: (a) liquid inside the shell is in rest; (b) liquid inside the shell is moving at constant speed. The frequency equations are constructed and are numerically realized in both cases. In the computation process, linear and parabolic laws have been adopted for the heterogeneity function. Specific curves are set up.

Keywords: Reinforced Shell, Orthotropic Shell, Variation Principle, Liquid, Free Oscillation.

I. INTRODUCTION

Resistance, oscillation, and strength analysis of thin-walled elements of shell type structures, dealing with the environment play an important role in the design of modern apparatus, machinery and facilities. The shell is reinforced by different ribs to give them greater rigidity. Such structures may be in contact with the liquid and be subject not only to static loads, but also dynamic. However, the behavior of heterogeneous, thin-walled elements of structures with ribs, consideration of their discrete location, the effects of liquid are not sufficiently explored. Therefore, the development of mathematical models for the study of the establishment of the reinforced heterogeneous orthotropic shells, which are best suited to their work in dynamic loads, and their research on sustainability and variability as well as the choice of rational design parameters contact with fluids, are relevant tasks [1].

It should be noted that the work [2, 3] is devoted to the study of the free oscillations of ribbed cylindrical shells filled with fluids. The influence of the number of ribs, their stiffness, the velocity of the fluid, the various mechanical, physical and geometric dimensions of the shell at the frequencies of their own vibrations and the optimization of the parameter of the circular ribbed cylindrical shell are studied.

The reference [4-6] deals with the study of the parametric oscillation of the non-linear and heterogeneous straight bar in the viscoelastic environment, using the Pasternak contact model. The influence of the main factors-elasticity of the base, the damageability material of the bar and the shell, constraints of the shear factor from the frequency of fluctuations in the longitudinal oscillation characteristics of the bar points in the viscoelastic environment are studied. In all the cases studied, dependence of the dynamic stability zone of the rod vibrations in the viscoelastic environment from the structure parameters on the load-frequency plane.

Reference [7] presents the results of a pilot study on the impact of reinforcing ribs and attached solids on the frequency and shape of free vibrations of subtle, structurally mixed shells. Frequency equations of ribbed cylindrical shells filled with fluid, approximate frequencies of the equation, and simple computational formulas to find the values of the minimum individual frequencies of the system reviewed were built using the asymptotic method and the forced fluctuations of the reinforced sheath, filled with fluid investigated and the amplitude-frequency characteristics of the reviewed oscillator processes defined in the reference [8, 9, 12, 13].

By entering a parameter determining the optimal reinforcement, the parameters of shells reinforced by the cross system of edges and filled liquid were optimized, and the influence of the degree of compressibility fluid on the frequency of the free axisymmetric of the oscillations of ribbed cylindrical shells was investigated.

II. PROBLEM STATEMENT

The ribbed shell is considered to be a system consisting of its own shell and tightly connected to the edges of the rib contact. It is assumed that the hard-deformable state of the shell can be fully defined within the linear theory of elastic thin shells, based on the Kirchhoff-Lyav hypothesis, and that the theory of curved bars Kirchhoff-Clebsch theory is applied for the calculation of the ribs. The coordinate system is selected so that the coordinate lines are coincident with the main curvature lines of the middle surface of the shell.

It also presumes that the edges are placed along the coordinate lines, and their edges, as well as the edges of the plating, lie in the same coordinate plane. It is also assumed that all edges form a regular system. The regular system of longitudinal and circular edges means a system in which the stiffness of all edges, their reciprocal distances equals, and the distances from the edge of the shell to the nearest edge equals the distance between edges.

The deformed state of the hull can be determined through the three components of its middle surface movement u, ϑ and w. In this case, the rotation angles of normal elements ϕ_1, ϕ_2 regarding position lines y and x expressed through w and ϑ using dependence $\phi_1 = -\frac{\partial w}{\partial x}$, $\phi_2 = -\left(\frac{\partial w}{\partial x} + \frac{\vartheta}{R}\right)$, where, R is radius of the middle surface of the shell.

It is noted that $h_i = 0.5 H_i^1$, where, h is shell thickness, H_i^1 is distances from axes, i is longitudinal bar till the shell surface, x_i and y_i are coordinates of the lines of conjugation of ribs with a shell, and ϕ_i, ϕ_{kpi} are angles of rotation and twisting of cross-sections of longitudinal rods.

With regard to the external effects, it is assumed that the surface loads for the ribbed shell from the side of the liquid, can be reduced to the constituent q_x, q_y and q_z, applied to the middle surface of the shell.

Differential equations of motion and the natural boundary conditions for the longitudinally reinforced, orthotropic, heterogeneous cylindrical shell with fluid will be derived from the variation principle of Hamilton-Ostrogradsky. This requires the prior record of the potential and kinetic energy of the system.

To accommodate heterogeneity, the thickness of the cylindrical shell will be based on the three-dimensional functionality. There are different ways to take into account the heterogeneity of the shell material. One of these is that the Young's modulus and the density of the shell material are accepted as normal coordinates z: $\tilde{E}_1 = \tilde{E}_1(z)$, $\tilde{E}_2 = \tilde{E}_2(z)$, $\rho = \rho(z)$. Poisson's ratio is assumed to be permanent. In this case, the full power of the cylindrical shell is in the form of:

$$V = \frac{1}{2}\iint \int_{-h/2}^{h/2} \left(\sigma_x e_x + \sigma_y e_y + \tau_{xy} e_{xy} + \rho(z)\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right) dx dy dz \tag{1}$$

where,

$$\sigma_x = b_{11}(z) e_x + b_{12}(z) e_y$$
$$\sigma_y = b_{12}(z) e_x + b_{22}(z) e_y \tag{2}$$
$$\sigma_{xy} = b_{66}(z) e_{xy}$$

$$e_x = \frac{\partial u}{\partial x}\ ;\ e_y = \frac{\partial \vartheta}{\partial y} + \frac{w}{R}\ ;\ e_{xy} = \frac{\partial u}{\partial y} + \frac{\partial \vartheta}{\partial x} \tag{3}$$

where,

$$b_{11}(z) = \frac{\tilde{E}_1(z)}{1-\nu_1\nu_2};\ b_{22}(z) = \frac{\tilde{E}_2(z)}{1-\nu_1\nu_2};$$

$$b_{12}(z) = \frac{\nu_2 \tilde{E}_1(z)}{1-\nu_1\nu_2} = \frac{\nu_1 \tilde{E}_2(z)}{1-\nu_1\nu_2};\ b_{66}(z) = G_{12}(z) = G(z)$$

Considering (2) and (3) in (1), we can write:

$$\begin{aligned} V = \frac{1}{2}\iint \int_{-h/2}^{h/2} \Bigg[& b_{11}(z)\left(\frac{\partial u}{\partial x}\right)^2 + \\ & + 2b_{12}(z)\left(\frac{\partial u}{\partial x}\frac{\partial \vartheta}{\partial y} + w\frac{\partial u}{\partial x}\right) + \\ & + b_{22}(z)\left(\left(\frac{\partial \vartheta}{\partial y}\right)^2 + 2w\frac{\partial \vartheta}{\partial y} + w^2\right) + \\ & + b_{66}(z)\left(\left(\frac{\partial u}{\partial y}\right)^2 + 2\frac{\partial u}{\partial y}\frac{\partial \vartheta}{\partial x} + \left(\frac{\partial \vartheta}{\partial x}\right)^2\right) + \\ & + \rho(z)\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \Bigg] dx dy dz \end{aligned} \tag{4}$$

Expressions for potential power of elastic deformation of i the longitudinal ribs are as follows [10]:

$$\begin{aligned} \Pi_i = \frac{1}{2}\int_0^L \Bigg[& \tilde{E}_i F_i \left(\frac{\partial u_i}{\partial x}\right)^2 + \tilde{E}_i J_{yi}\left(\frac{\partial^2 w_i}{\partial x^2}\right)^2 + \\ & + \tilde{E}_i F_{zi}\left(\frac{\partial^2 \vartheta_i}{\partial x^2}\right)^2 + \tilde{G}_i J_{kpi}\left(\frac{\partial \phi_{kpi}}{\partial x}\right)^2 \Bigg] dx \end{aligned} \tag{5}$$

The kinetic energy of the edges is written in the form:

$$\begin{aligned} K_i = \rho_i F_i \int_{x_1}^{x_2} \Bigg[& \left(\frac{\partial u_i}{\partial t}\right)^2 + \left(\frac{\partial \vartheta_i}{\partial t}\right)^2 + \left(\frac{\partial w_i}{\partial t}\right)^2 + \\ & + \frac{J_{kpi}}{F_i}\left(\frac{\partial \phi_{kpi}}{\partial t}\right)^2 \Bigg] dx \end{aligned} \tag{6}$$

In the Equations (4) and (6) F_i, J_{zi}, J_{yi}, J_{kpi} are area and moments of inertia of the cross section of i longitudinal bar, respectively, relative to the axis oz and axis, parallel axis oy and passing through the center of gravity of the section as well as its moment of inertia in torsion; $\tilde{E}_i, \tilde{G}_i$ are moduli of elasticity and shear of the material of longitudinal i temporal coordinate t, and ρ_i is according to the density of the materials produced longitudinal rod i.

The potential energy of external surface loads acting on the side of the ideal fluid applied to the shell is defined as the work performed by these loads when the system is transferred from the deformed state to the initial undistorted condition, and is presented as:

$$A_0 = -\int_0^L \int_0^{2\pi} q_z w dx dy \tag{7}$$

The total energy of the system is equal to the sum of the energy of the elastic deformation of the shell and transverse ribs, as well as the potential energies of all external loads acting on the side of the perfect fluid:

$$J = V + \sum_{i=1}^{k_1} \left(\Pi_i + K_i\right) + A_0 \tag{8}$$

where k_1 is the number of longitudinal ribs. Assuming that the primary velocity of the flow is equal to U and deviations from this speed are small, use the wave equation for the potential of indignant speeds ϕ by [11]:

$$\Delta\tilde{\phi} - \frac{1}{a_0^2}\left(\frac{\partial^2\tilde{\phi}}{\partial t^2} + 2U\frac{\partial^2\tilde{\phi}}{R\partial\xi\partial t} + U^2\frac{\partial^2\tilde{\phi}}{R^2\partial\xi^2}\right) = 0 \tag{9}$$

Full energy expression of the system (8), the equation of fluid motion (9) is supplemented by contact conditions. On the contact surface, the shell-fluid is observed to be the continuity of the radial velocities and pressures. The condition of impermeability or fluidity at the wall of an environment has a form [11]:

$$\vartheta_r\big|_{r=R} = \frac{\partial\phi}{\partial r}\bigg|_{r=R} = -\left(\omega_0\frac{\partial w}{\partial t^1} + U\frac{\partial w}{R\partial\xi}\right) \tag{10}$$

The equality of radial pressure from the liquid to shell:

$$q_z = -p\big|_{r=R} \tag{11}$$

The frequency equation of a ribbed inhomogeneous shell with a flowing liquid is obtained on the basis of the stationarity principle of Hamilton-Ostrogradsky action:

$$\delta W = 0 \tag{12}$$

where, $W = \int_{t'}^{t''} J dt$ is Hamilton's action, t' and t'' are arbitrary moments of time.

Supplementing the full power of the system with contact conditions (8), equations of fluid motion (9) we reach to the problem of natural oscillations of a longitudinally supported heterogeneous orthotropic cylindrical shell with a flowing liquid. In other words, the challenge of its own fluctuations in the longitudinally orthotropic cylindrical shell with the flowing fluid is the joint integration of expressions for full energy of the system (8), equation of Fluid Motion (9) under the conditions (10) and (11) on the surface of their contact.

III. PROBLEM SOLUTION

Potential for indignant speed ϕ we are searching in the form:

$$\phi(\xi, r, \theta, t_1) = f(r)\cos n\phi \sin\chi\xi \sin\omega_1 t_1 \tag{13}$$

Using (10) from the condition (7) and (8) we have:

$$\tilde{\phi} = -\Phi_{\alpha n}\left(\omega_0\frac{\partial w}{\partial t_1} + U\frac{\partial w}{R\partial\xi}\right)$$
$$p = \Phi_{\alpha n}\rho_m\left(\omega_0\frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0\frac{\partial^2 w}{R\partial\xi\partial t_1} + +U^2\frac{\partial^2 w}{R^2\partial\xi^2}\right) \tag{14}$$

where,

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r)/I_n'(\beta r) & , M_1 < 1 \\ I_n(\beta_1 r)/J_n'(\beta_1 r) & , M_1 > 1 \\ \dfrac{R^n}{nR^{n-1}} & , M_1 = 1 \end{cases} \tag{15}$$

where, $M_1 = \dfrac{U + \omega_0 R\omega_1/\alpha}{a_0}$, $\beta^2 = R^{-2}\left(1 - M_1^2\right)\chi^2$, $\beta_1^2 = R^{-2}\left(M_1^2 - 1\right)\chi^2$, I_n is the modified Bessel function of the first kind n, J_n is Bessel function of the first kind n, and $\omega_0 = \sqrt{\dfrac{E_0}{\left(1-\nu^2\right)\rho_0 R^2}}\omega_1 = \omega/\omega_0$.

We will search for the movement of the shell in the form:

$$u = u_0 \sin\chi\xi \cos n\theta \sin\omega_1 t_1$$
$$\vartheta = \vartheta_0 \cos\chi\xi \sin n\theta \sin\omega_1 t_1 \tag{16}$$
$$w = w_0 \cos\chi\xi \cos n\theta \sin\omega_1 t_1$$

where, u_0, ϑ_0, w_0 are unknown constants; and χ, n are wave numbers in the longitudinal and district directions.

Using (4), (7) and (11) the task amounts to homogeneous system of linear algebraic equations of the third order.

$$a_{i1}u_0 + a_{i2}\vartheta_0 + a_{i3}w_0 = 0, \ (i = 1,2,3) \tag{17}$$

where, the elements $a_{i1}, a_{i2}, a_{i3}\ (i = 1,2,3)$ are unwieldy, so they are not listed here. The non-trivial solution of the system of linear algebraic Equations (17) is possible only if when ω_1 is root of its determinant. Determination ω_1 boils down to the transcendental equation, since ω_1 included in the arguments of the Bessel function:

$$\det a_{ij} = 0, \ i, j = 1,3 \tag{18}$$

IV. NUMERICAL RESULTS

The frequency Equation (18) was solved numerically with the following initial data:

$R = 160$ mm; $\tilde{E}_i = 6.67\times10^9 \frac{\text{n}}{\text{m}^2}$; $h = 0.45$ mm; $L = 800$ mm; $\rho_i = 7.8$ q/cm^3; $b_{11} = 18.3\Gamma\Pi a$; $b_{12} = 2.77\Gamma\Pi a$; $b_{22} = 25.2\Gamma\Pi a$; $b_{66} = 3$; $F_i = 5.75$ mm^2; $I_{xj} = 19.9$ mm^4; $al_0 = 1430$ m/sec; $\frac{F_i}{2\pi Rh} = 0.1591\times10^{-1}$; $\frac{I_{kpi}}{2\pi R^3 h} = 0.5305\times10^{-6}$; $\frac{F_i}{2\pi Rh} = 0.1591\times10^{-1}$.

Two types of laws of variations in inhomogeneity are considered as
Linear:

$$\tilde{E}_i(z) = \tilde{E}_{0i}\left[1+\alpha\left(\frac{z}{h}\right)\right] \quad , \quad \rho(z) = \rho_0\left[1+\alpha\left(\frac{z}{h}\right)\right] \quad \text{and}$$

Parabolic:

$$\tilde{E}_i(z) = \tilde{E}_{0i}\left[1+\alpha\left(\frac{z}{h}\right)^2\right] \quad , \quad \rho(z) = \rho_0\left[1+\alpha\left(\frac{z}{h}\right)^2\right],$$

where $\tilde{E}_{0i}\,(i=1,2)$ is Young's Modulus, and α is Parameter Heterogeneity. Note that, in linear law, the change $|\alpha|<1$, at the parabolic change α is arbitrary.

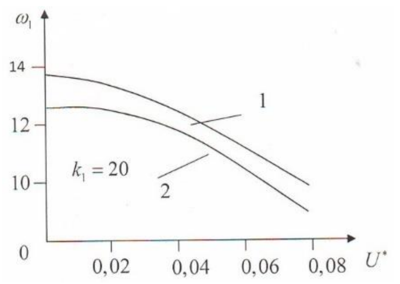

Figure 1. The dependence of the frequency parameter on the speed of the fluid: 1-linear law, 2-parabolic law

The results of the calculation are shown in Figures 1 and 2. Figure 1 shows the constraints of the frequency parameter ω_1 from the relative velocity of the flow U^* for different laws of heterogeneity variations in the shell thickness. It shows that an increase in speed leads to a decrease in frequency.

It should be noted that $U^*=0$ corresponds to rest fluid. Figure 2 illustrates the influence of the number of longitudinal ridge k_1 on the frequency parameters ω_1 fluctuations in the system. It is clear with the increase k_1 frequency parameters ω_1 the oscillations of the system are increased at first, and then at a certain value k_1 begin decreasing. Due to the fact that with the increase of k_1 rod weight increases and this results in a significant impact of their inertial properties on the fluctuation process.

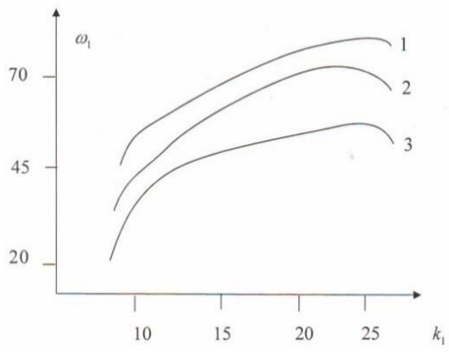

Figure 2. The dependency of the frequency parameter on the number of longitudinal edges: 1-Homogeneous shell, 2-linear law, 3-parabolic law

The comparison of the graphs shows that the accounting for heterogeneity results in lowering the values of the system's own fluctuation frequencies compared to the same system when the shell is homogeneous.

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BIOGRAPHIES

Fuad Seyfeddin Latifov was born in Ismayilly, Azerbaijan, in 1955. He graduated from Faculty of Mechanics-Mathematics, Azerbaijan State University, Baku, Azerbaijan in 1977. In 1983, he received his M.Sc. degree in Physics-Mathematics from Saint Petersburg State University, Saint Petersburg, Russia. In 2003, he got the Ph.D. degree in Physics-Mathematics. He is a Professor and the Chief of Department of Higher Mathematics in Azerbaijan University of Architecture and Construction Baku, Azerbaijan. He has written more than 80 scientific articles and 11 monographs. He is the coauthor of an encyclopedia on mathematics.

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