International Journal on "Technical and Physical Problems of Engineering" (IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

December 2019 | Issue 41 | Volume 11 | Number 4 | Pages 16-21

PROPAGATION CHARACTERISTICS OF NONLINEAR LOW FREQUENCY ELECTROSTATICS MODES IN QUANTUM PLASMA

N. Rani M.K. Yadav

Department of Physics, J.C. Bose University of Science and Technology, YMCA, Faridabad, India
neelamraheja344@gmail.com, yadavmanikant64@gmail.com

Abstract- We investigated the propagation characteristics of nonlinear low frequency electrostatic waves in electron-ion quantum plasma. After reviewing the basic introduction of quantum plasma, we describe the nonlinear phenomenon of electrostatic wave. We presented that the electrons are degenerate and weakly correlated, whereas ions are non-degenerate and strongly correlated. The reductive perturbation technique is employed for weakly nonlinear electrostatic modes. In this study, we observed that the ion viscous dissipation affects the weakly nonlinear structures due to ion-ion correlations. In this research work, our emphasis on the dispersion relation and shown the existence of shock wave in dissipation dominated due to ion-ion correlation in the weakly nonlinear limit.

Keywords: Electrostatic Wave, Quantum Plasma, KdVB (Korteweg-de Vries Burger) Equation.

I. INTRODUCTION

In recent years, there has been the rapidly growing interested in global properties of quantum plasma in field of modern sciences and technologies in compact of astrophysical objects, such as white-dwarfs, neutron stars, and pulsars, etc. Actually, all plasmas are in some sense are quantum because it consists of charged particles as they obey the laws of quantum mechanics [1]. Although the density of classical plasma increases or its temperature decreases, it can enter a region where quantum effect starts. Quantum plasmas are obtained in high density matter. Dense plasma can be explained as the collective behavior of charged particle in which electrons are degenerate and weakly correlated whereas ions are non-degenerate and strongly correlated [2].

The nonlinear interaction between matter and wave in plasmas is the most important topic of plasma physics. For modern device physicists dealing with quantum wells, quantum wires, quantum dots, etc. the linear and nonlinear behaviors of waves and instabilities, through the carrier's dynamics in semiconductors, are crucially important [3]. In quantum plasma fluids, theoretical investigation of nonlinear phenomena associated with electrostatic waves has been carried out by the number of authors.

In quantum plasmas, the Collective interactions between an ensemble of degenerate electrons and positrons/holes give rise to novel waves and structures by Bohm and Pines in 1953 [4, 5]. The basic concept of semiconductor quantum plasma is the de Broglie wavelengths of the plasma particles may be comparable to the Debye length [6] or other scale lengths of the plasma by using magneto hydrodynamic (MHD) model for plasmas, have developed quantum hydrodynamic (QHD) model to study the quantum corrections in plasma characteristics by Haas [7], Manfredi [8] and M. Marklund, P.K. Shukla in 2006 [9]. In quantum plasmas, due to inter-fermion distances much lower than its de Broglie wavelength and the influence of the Pauli exclusion rule, many quantum effects such as electron-tunneling, degeneracy pressure, and Landau quantization may occur.

Quantum plasma shows dispersion instead of dissipation, which is caused by quantum tunneling effects described by Bohm potential term. Whereas dissipation may arise due to kinematic of viscosity, collisions and wave is propagation is governed by interplay between the quantum tunneling and wave particle interactions.

Most of these works have based on quantum hydrodynamic (QHD) model of plasmas. This model is very useful to study the short-scale collective phenomena, such as waves, instabilities, linear, and nonlinear interactions in dense plasmas [10, 12]. By the quantum hydrodynamics model (QHM), the low frequency electrostatic modes have been investigated in weakly coupled quantum plasma. In dusty plasmas, the low frequency dust acoustic waves is strongly correlated in classical plasmas with non-degenerate electrons, ions and charge dust grains have been observed. The nonlinear studies of electrostatic and electromagnetic waves in quantum plasma were based on the generalized quantum hydrodynamical (GQHD) equations [13, 15] for nonrelativistic degenerate electron fluids supplemented by Poisson's and Maxwell's equations.

In this paper, the electrons are degenerate and weakly correlated is described by quantum hydrodynamic model (QHM), whereas ions are non-degenerate and strongly correlated is described by generalized hydrodynamic model (GHM).

In this research work, our main emphasis on dispersion relation for nonlinear electrostatic modes in strongly coupled plasmas by using continuity equations and Poisson's equation. In weakly nonlinear limit, ion-ion correlation effects introduce a viscous dissipation, which is responsible for Korteweg-de Vries Burger equation. The dispersion relation is analyzed theoretically and numerically, and this solution shows existence the shock wave in dissipation dominated.

II. THEORETICAL FORMULATION

In order to study the weakly nonlinear low frequency electrostatic wave propagation characteristics, we take the assumption to solve our problem.

1. The degeneracy parameter for a particle kind *a* is defined as:

$$\chi_a = \frac{\varepsilon_{F_a}}{T_a} = \frac{1}{2}\left(3\pi^2\right)^{2/3}\left(n_{a0}\lambda_{B_a}^3\right)^{2/3}$$

where, $\varepsilon_{F_a} = \hbar^2\left(3\pi^2 n_{a0}\right)^{2/3}/2m_a$ are the Fermi energy of ground state and $\lambda_{B_a} = \hbar/\sqrt{m_a T_a}$ is the thermal de Broglie wavelength of particle kind *a*.

(i) The electrons are fully degenerate so that the electron Fermi energy (ε_{Fe}) is much larger than electron thermal energy (T_e) and electron degeneracy parameter $\chi_e >> 1$. This shows that

$$n_{e0} >> \frac{1}{3\pi^2}\left(\frac{2m_e T_e}{\hbar^2}\right)^{3/2} = n_{eQ}$$

(ii) The ions are non-degenerate so that ion Fermi energy (ε_{Fe}) is much smaller than ion thermal energy (T_i) and ion degeneracy parameter $\chi_e << 1$. This shows that

$$n_{i0} << \frac{1}{3\pi^2}\left(\frac{2m_i T_i}{\hbar^2}\right)^{3/2} = n_{iQ}$$

2. The electron correlations are neglected as electron-electron correlation effects are negligibly small compared to ion-ion correlation. The electron coupling parameter is

$$\Lambda_e = \left(\frac{1}{n_{e0}\lambda_{Fe}^3}\right)^{2/3} = 1.5Z^{-5/3}\chi_e^{-1}\varphi$$

The electron degeneracy parameter $\chi_e >> 1$, $Z \geq 1$ and $\varphi = (T_i/T_e) \leq 1$. Where $\lambda_{Fe} = v_{Fe}/\sqrt{3}\omega_{pe}$ is the Thomas-Fermi three dimensional screening length of electrons, $\omega_{pe} = \sqrt{\frac{n_{e0}e^2}{\varepsilon_0 m_e}}$ is the electron plasma frequency and $v_{Fe} = \sqrt{\frac{2\varepsilon_{Fe}}{m_e}}$ is the Fermi speed of electrons. The electron-ion interactions are weak compared to ion-ion correlations and, therefore, we neglect electron-ion interactions.

3. The ions are strongly correlated, i.e., ion coupling parameter is:

$$\Lambda_i = \left(\frac{1}{n_{i0}\lambda_{Di}^3}\right)^{2/3} = \frac{Z^2e^2}{\varepsilon_0 x_i T_i} >> 1$$

where, $\lambda_{Di} = \sqrt{\varepsilon_0 T_i}/(n_{i0}Z^2e^2)$ is the ion Debye radius and $x_i = \left(3/4\pi n_{i0}\right)^{1/3}$ is inter ionic distance. This implies that:

$$n_{i0} >> \frac{3}{4\pi}\left(\frac{Z^2e^2}{\varepsilon_0 T_i}\right)^{-3} = n_{SC}$$

The assumptions 1 and 3 determine highly dense quantum plasma system [$n_{eQ} \ll n_{e0}$, $n_{SC} << n_{i0} << n_{iQ}$]. Our assumptions are valid for a typical hydrogen plasma if plasma number density $n_{i0} \sim \left(10^{28} - 10^{32}\right)\, m^{-3}$ and $\varphi = (T_i/T_e) = 1$. It means physical plasma system is highly dense if the ion coupling parameter $\Lambda_i >> 1$ (strongly coupled) in which electrons with weak interactions form a degenerate quantum fluids whereas ions with strong interactions form a classical fluids.

In order to account for the correlation among ion dynamics, we use viscoelastic approach which is described by general hydrodynamics model [16, 17]. We consider generalized momentum equation for ion fluid using the relation $\frac{d}{dt} = \frac{\partial}{\partial t} + u_i.\nabla$ is:

$$\left(1+\tau_m\frac{d}{dt}\right)\left[\rho_i\frac{du_i}{dt} + \nabla P_i - Zen_i\vec{E}\right] = \\ = \eta\nabla^2 u_i + \left(\kappa + \frac{\eta}{3}\right)\nabla\left(\nabla.u_i\right) \quad (1)$$

where, τ_m is the viscoelastic relaxation time that accounts the memory function, u_i is ion fluid velocity, $\rho = m_i n_i$ is ion mass density, P_i is ion pressure, η and κ are the shear and bulk coefficients of viscosity.

The general hydrodynamic model also includes ion continuity equation and energy equation. The ion energy equation is not required because ion dynamics is isothermal at strong couplings.

$$\frac{\partial n_i}{\partial t} + \nabla(n_i u_i) = 0 \quad (2)$$

the gradient ion pressure becomes:

$$\nabla P_i = T_i\mu_i\nabla n_i$$

where, μ_i is the coefficient of isothermal compressibility for ion fluid.

The conservation of momentum equation for electron is:

$$0 = -e\vec{E} - \frac{\nabla P_e}{n_e} + \frac{\hbar^2}{2m_e}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) \quad (3)$$

where, n_e is unperturbed electron number density, P_e is electron pressure and the term $\hbar^2$ arises due to electron tunneling through the Bohm potential [18]. The system of equations is closed by Poisson's equation.

$$\nabla . E = \frac{e}{\varepsilon_0}\left(Z n_i - n_e \right) \tag{4}$$

In strongly coupled quantum plasma, we considered the ion viscous dissipation effects the weakly nonlinear structures in 1-dimensional in the hydrodynamic range $(\omega \tau_m << 1)$.

To explore the nonlinear structures, it is convenient to write governing equations in dimensionless form. We use following dimensionless variables:

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \mu_i \frac{\partial \ln n_i}{\partial x} - \vec{E} = \frac{\eta^*}{n_i}\frac{\partial^2 u_i}{\partial x^2} \tag{5}$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}\left(n_i u_i \right) = 0 \tag{6}$$

$$\frac{\partial \vec{E}}{\partial x} = n_i - n_e \tag{7}$$

where, $\bar{H} = \frac{H}{3} = \frac{\hbar \omega_{pe}}{2T_{Fe}}$ the quantum diffraction due to 1-dimensional electron tunneling effect and $\bar{\lambda}_{Fe} = \sqrt{3}\lambda_{Fe} = v_{Fe} / \omega_{pe}$ is the Thomas-Fermi 1-dimensional screening length of electrons.

Now, we derive the Korteweg-de Vries equation from (5)-(7) by employing the reductive perturbation technique and the stretched coordinates:

$$\delta = \varepsilon^{1/2}\left(x - Mt \right) \tag{8}$$

$$\tau = \varepsilon^{3/2} t \tag{9}$$

where, ε is a smallness parameter proportional to the amplitude of the perturbation and M is the mode normalized by the ion thermal speed.

We can expand the variables $n_{e(i)}$, u_i and E in a power series of ε as:

$$n_{e(i)} = 1 + \varepsilon n_{e(i)}^{(1)} + \varepsilon^2 n_{e(i)}^{(2)} + ... \tag{10}$$

$$u_i = 0 + \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + ... \tag{11}$$

$$\vec{E} = 0 + \varepsilon^{3/2} E^{(1)} + \varepsilon^{5/2} E^{(2)} + ... \tag{12}$$

Now, using (10)-(12) in (5)-(8) and taking the coefficient of $\varepsilon^{3/2}$ from (12) and ε from (10)-(11), we get:

$$E^{(1)} = \frac{\partial\left(\mu_i n_i^{(1)} - M u_i^{(1)} \right)}{\partial \delta} \tag{13}$$

$$u_i^{(1)} = M n_i^{(1)} \tag{14}$$

$$E^{(1)} = -\left(\frac{\bar{\lambda}_{Fe}}{\lambda_{Di}} \right)^2 \frac{\partial n_e^{(1)}}{\partial \delta} \tag{15}$$

$$n_e^{(1)} = n_i^{(1)} \tag{16}$$

Now, using (13)-(16) we have:

$$M = \sqrt{\mu_i + \left(\frac{\bar{\lambda}_{Fe}}{\lambda_{Di}} \right)^2} \tag{17}$$

Now substituting (13)-(16) into (5)-(7) and equating the coefficient of from (10)-(12), we obtain:

$$E^{(2)} = \frac{\partial u_i^{(1)}}{\partial \tau} + u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \delta} - \mu_i n_i^{(1)} \frac{\partial n_i^{(1)}}{\partial \delta} - \eta^* \frac{\partial^2 u_i^{(1)}}{\partial \delta^2} + \mu_i \frac{\partial n_i^{(2)}}{\partial \delta} - M \frac{\partial u_i^{(2)}}{\partial \delta} \tag{18}$$

$$\frac{\partial n_i^{(1)}}{\partial \tau} + \frac{\partial\left(n_i^{(1)} u_i^{(1)} \right)}{\partial \delta} = M \frac{\partial n_i^{(2)}}{\partial \delta} - \frac{\partial u_i^{(2)}}{\partial \delta} \tag{19}$$

$$E^{(2)} = -\left(\frac{\bar{\lambda}_{Fe}}{\lambda_{Di}} \right)^2 n_e^{(1)} \frac{\partial n_e^{(1)}}{\partial \delta} - \left(\frac{\bar{\lambda}_{Fe}}{\lambda_{Di}} \right)^2 \frac{\partial n_e^{(2)}}{\partial \delta} + \frac{\bar{H}^2}{4}\left(\frac{\bar{\lambda}_{Fe}}{\lambda_{Di}} \right)^4 \frac{\partial^3 n_e^{(1)}}{\partial \delta^3} \tag{20}$$

$$\frac{\partial E^{(1)}}{\partial \delta} = n_i^{(2)} - n_e^{(2)} \tag{21}$$

Now, using above equation and eliminating $n_{e(i)}$, u_i and E, we obtain:

$$\psi = \frac{M n_i^{(1)}}{\mu_i + 2\left(\frac{\bar{\lambda}_{Fe}}{\lambda_{Di}} \right)^2} \tag{22}$$

$$\frac{\partial \psi}{\partial \tau} + \psi \frac{\partial \psi}{\partial \delta} + A \frac{\partial^3 \psi}{\partial \delta^3} = \mu \frac{\partial^2 \psi}{\partial \delta^2} \tag{23}$$

where, the coefficients A and μ are given by

$$A = \frac{1}{2M}\left(\frac{\bar{\lambda}_{Fe}}{\lambda_{Di}} \right)^2 \left(1 - \frac{\bar{H}^2}{4} \right), \quad \mu = \frac{\eta^*}{2} \tag{24}$$

Equation (23) is the Korteweg-de Vries Burger equation of the weakly nonlinear low frequency electrostatic wave in strongly coupled quantum plasma. The solution of Korteweg-de Vries equation is found by transforming the independent variables δ and τ to:

$$K = \delta - C_0 \tau, \; \tau = \tau \tag{25}$$

where, C_0 is a constant velocity normalized by c.

III. NUMERICAL SOLUTION AND DISCUSSION

In order to get the shock structure, it is necessary to apply the boundary condition on wave. The exact solution of KDVB is not possible because this equation is not exactly integral solution. A particular solution of KDVB is possible, which are only for monotonic shock structure. Actually, a monotonic shock structure is possible only when dissipation dominates, and oscillatory shock structure is possible only when dissipation is weak. The boundary condition is:

$$\psi \to 0, \; \frac{d\psi}{dK} \to 0, \; \frac{d^2\psi}{dK^2} \to 0 \text{ at } K \to \infty$$

Finally, Equation (22) becomes:

$$\frac{d^2\psi}{dK^2} = \left(\frac{C_0}{A} \right)\psi - \left(\frac{1}{2A} \right)\psi^2 + \left(\frac{\mu}{A} \right)\frac{d\psi}{dK} \tag{26}$$

Equation (26) is well known as the damped harmonic oscillator in which ψ describes the generalized coordinate and δ describes the time. Equation (26) has two singular points:

(i) $\psi \to 0,\ \dfrac{d\psi}{dK} \to 0$

(ii) $\psi \to 2C_0,\ \dfrac{d\psi}{dK} \to 0$

The first point shows the equilibrium downstream state and the second point shows the upstream state. If we assume, $K = \infty(\delta = \infty)$, the particle was located at $\psi = 0$ and if $K = -\infty(\delta = -\infty)$, the particle at point $\psi = 2C_0$. The shock strength is:

$Shock\ strength = \psi(-\infty) - \psi(+\infty) = 2C_0$

The Mach number is independent of dispersion:

$MA = Nonlinear\ Wave\ Velocity / Linear\ Wave\ Velocity$

$$MA = 1 + \varepsilon\left(\frac{C_0}{M}\right) \tag{27}$$

To get the nature of shock structure, the solution of Equation (26) is obtained by substitute the $\psi = 2C_0 + \bar{\psi}$, where $2C_0 >> \bar{\psi}$.

$$\frac{d^2\bar{\psi}}{dK^2} + \left(\frac{C_0}{A}\right)\bar{\psi} - \left(\frac{\mu}{A}\right)\frac{d\bar{\psi}}{dK} = 0 \tag{28}$$

The solution of equation (28) is proportional to ~ exp (σK), where:

$$\sigma = \frac{\mu}{2A} \pm \sqrt{\left(\frac{\mu^2}{4A^2} - \frac{C_0}{A}\right)} \tag{29}$$

$\bar{H} = 0.2$

The oscillatory shock structure in which dispersion dominates over dissipation by different values of τ as shown in Figure 1 and this follows the Equation (29) with the singular point $(2C_0,\ 0)$. It is clear from Figure 1(d), the oscillatory shock is fully developed at $\tau = 1200$ with singular point $2C_0 = 0.1$ giving shock speed $C_0 = 0.05$, dispersion coefficient A=3 and burger coefficient $\mu = 10^{-2}$.

$$\psi = C_0\left[1 - \tanh\left(\frac{C_0 K}{2\mu}\right)\right] \tag{30}$$

$\bar{H} = 2$

The monotonic shock structure in which dissipation dominates over dispersion by different values of τ as shown in Figure 2 and this follows the Equation (30) with the singular point $(2C_0,\ 0)$. In Figure 2, the dispersion coefficient $A = 0$, C_0 is the amplitude and $2\mu / C_0$ is width of the shock and other values are same as in Figure 1. In this range, burger coefficient $\mu = 10^{-2}$, dispersion coefficient $A = 1 \sim 3$ and $0 < H < 2$ [19, 20], monotonic shock structure is well agree with Equation (30).

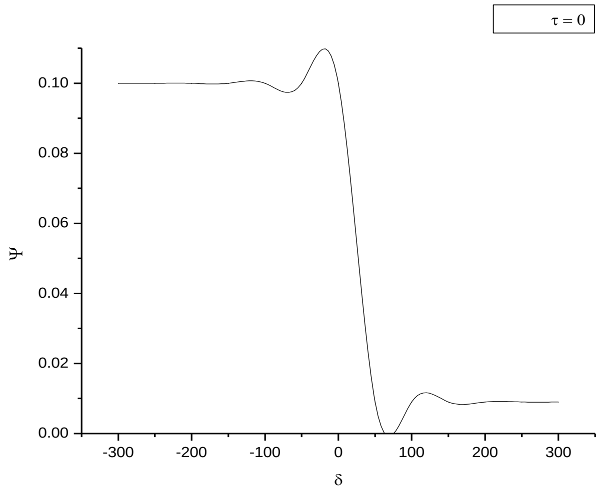

Figure 1(a). Oscillatory shock structure at τ=0

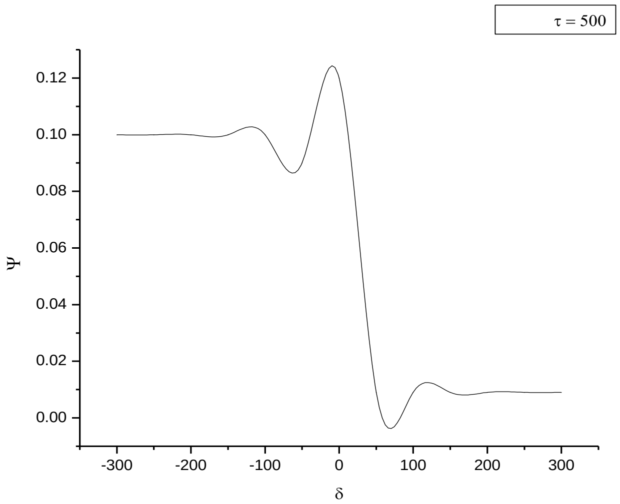

Figure 1(b). Oscillatory shock structure at τ=500

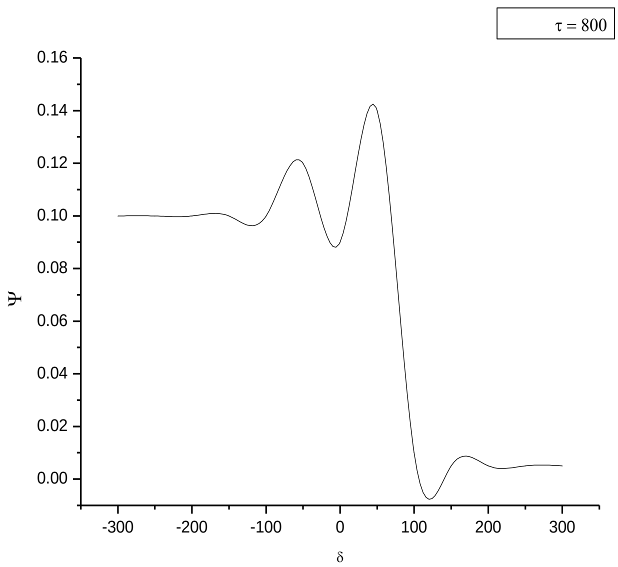

Figure 1(c). Oscillatory shock structure at τ=800

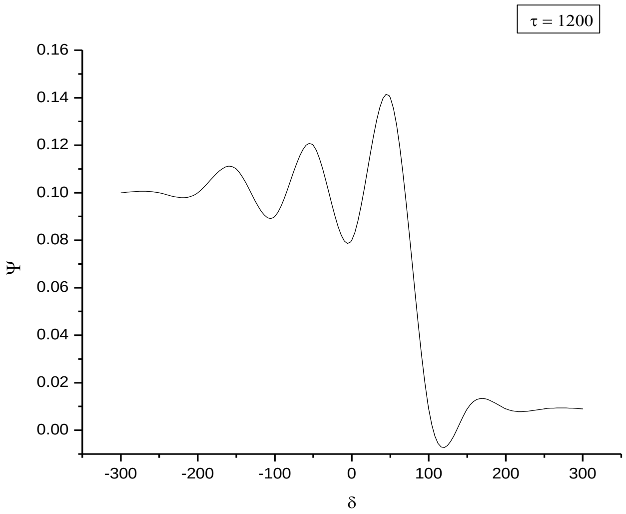

Figure 1(d). Oscillatory shock structure at τ=1200

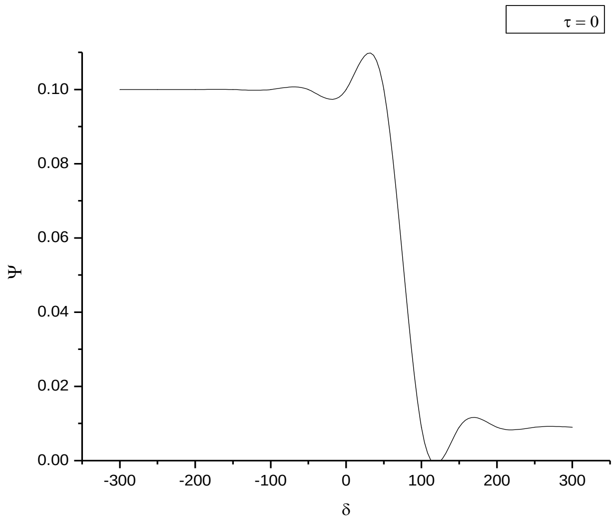

Figure 2(a). Monotonic shock structure at τ=0

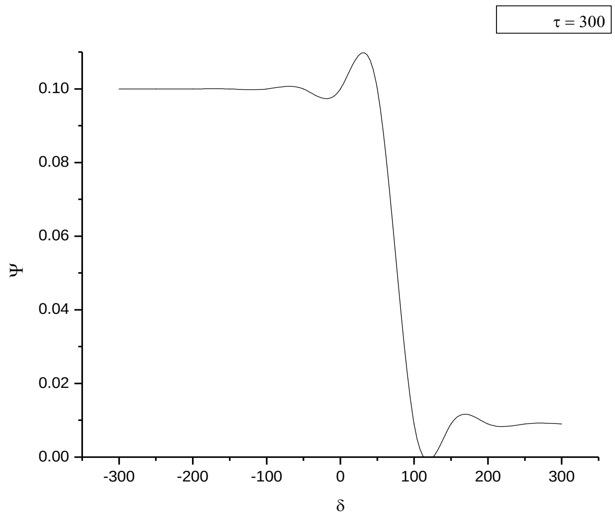

Figure 2(b). Monotonic shock structure at τ=300

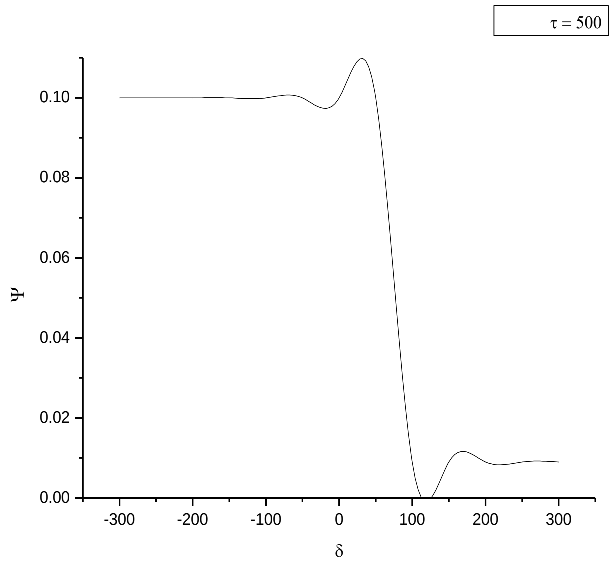

Figure 2(c). Monotonic shock structure at τ=500

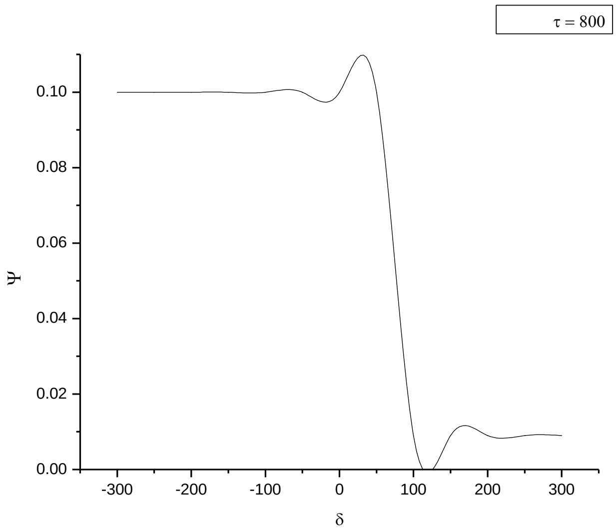

Figure 2(d). Monotonic shock structure at τ=800

IV. CONCLUSION

The propagation of nonlinear low frequency electrostatic modes in strongly coupled quantum plasma has investigated. The behavior of strongly coupled quantum plasma is the collective nature, which is most important property of this plasma. This investigation supports the existence of shock wave due to ion-ion correlation in high energy density of strongly coupled quantum plasma. The oscillatory shock structure in which dispersion dominates over dissipation and monotonic shock structure in which dissipation dominates over dispersion by different values of τ are discussed. The results may be significant for understanding the scattering process involving intense laser beam in high energy density compressed plasma experimentally. In dissipative plasma, the propagation of small but finite amplitude nonlinear excitations maybe described by Korteweg-de Vries Burger equation.

REFERENCES

[1] A. Agrawal, N. Yadav, S. Ghosh, "Diffusion Induced Modulational Instability in Semiconductor Quantum Plasma with Strain Dependent Dielectric Constant", IJESRT, Vol. 6, pp. 381-389, 2017.
[2] P.K. Shukla, B. Eliasson, "Colloquium: Nonlinear Collective Interactions in Quantum Plasmas with Degenerate Electron Fluids", Reviews of Modern Physics, Vol. 83, pp. 885-909, 2011.
[3] S. Rehman, "Linear and Nonlinear Quantum Ion Acoustic Waves in a Plasma with Positive, Negative Ion and Fermi Electron Gas", Physics of Plasma, Vol. 17, p. 062303, 2010.
[4] D. Bohm, D. Pines, "A Collective Description of Electron Interactions; III. Coulomb Interactions in a Degenerate Electron Gas", Physical Review, Vol. 92, pp. 609-625, 1953.
[5] Ch. Uzma, I. Zeba, H.A. Shah, M. Salimullah, "Stimulated Brillouin Scattering of Laser Radiation in a Piezoelectric Semiconductor: Quantum Effect", Journal of Applied Physics, Vol. 105, p. 013307, 2009.
[6] R. Vanshpal, S. Dubey, S. Ghosh, "Quantum Effect on Parametric Dispersion in Presence of Nonuniform Size Colloids in Semiconductors", Journal of Physics: Conference Series, Vol. 365, pp. 0120461-0120465, 2012.
[7] F. Haas, "Quantum Magnetohydrodynamics", Physics of Plasmas, Vol. 12, p. 062117, 2005.
[8] G. Manfredi, "How to Model Quantum Plasmas", Field Inst. Commun., Vol. 46, pp. 263-287, 2005.
[9] M. Marklund, P.K. Shukla, "Nonlinear Collective Effects in Photon-Photon and Photon-Plasma Interactions", Reviews of Modern Physics, Vol. 78, p. 591, 2006.
[10] F. Hass, A. Bret, "Nonlinear Low Frequency Collisional Quantum Buneman Instability", Europhysics Letter, Vol. 97, p. 26001, 2012.
[11] S. Ghosh, S. Dubey, R. Vanshpal, "Steady State and Transient Gain Characteristics of the Stimulated Brillouin Scattered Mode in Quantum Semiconductors Plasma", Chinese Journal of Physics, Vol. 51, pp. 1251-1269, 2013.
[12] M.R. Amin, "Astrophysics and Space Science", Springer, Vol. 359, 2015.
[13] H.F. Wilhelm, Z. Physics, Vol. 1, p. 241, 1971.
[14] C.L. Gardner, C. Ringhofer, "Smooth Quantum Potential for the Hydrodynamic Model", Physical Review E, Vol. 53, p. 157, 1996.
[15] G. Manfredi, F. Haas, "Self-Consistent Fluid Model for a Quantum Electron Gas", Physical Review B, Vol. 64, pp. 075316, 2001.
[16] P.K. Kaw, A. Sen, "Low Frequency Modes in Strongly Coupled Dusty Plasmas", Physics of Plasmas, Vol. 5, p. 3552, 1998.
[17] M.S. Janaki, N. Chakrabarti, "Shear Wave Vortex Solution in a Strongly Coupled Dusty Plasmas", Physics of Plasmas, Vol. 17, pp. 053704-1-053704-3, 2010.
[18] P.K. Shukla, B. Eliasson, L. Stenflo "Stimulated Scattering of Electromagnetic Waves Carrying Orbital Angular Momentum in Quantum Plasmas", Physical Review E, Vol. 86, pp. 016403-1-016403-5, 2012.
[19] V.I. Karpman, "Nonlinear Waves in Dispersive Media", Elsevier, 1st Edition, Pergamon, London, 1975.
[20] A. Jeffery, T. Kakutani, "Weak Nonlinear Dispersive Wave: A Discussion Centered Around the Korteweg - De Vries Equation", SIAM Rev, Vol. 14, pp. 582-643, 1972.

BIOGRAPHIES

Neelam Rani was born in Haryana, India in 1981. She received the B.Sc. degree from Kurukshetra University, Kurukshetra, India in 2002 and the M.Sc. degree from Guru Jambheshwar University of Science & Technology, Hisar, India in 2004 and M. Phil. degree from Chaudhary Devi Lal University, Sirsa, India in 2007. Currently, she is Research Scholar of Physics at J.C. Bose University of Science & Technology, YMCA, Faridabad, India. She has published several research papers in the field of plasma physics. She is author of one book of engineering physics. Her research interests are in the area of plasma physics and nonlinear waves.

Mani Kant Yadav was born in Bihar, India in 1966. He received the B.Sc. and M.Sc. degrees from Lalit Narayan Mithila University, Darbhanga, India in 1987 and 1991, respectively and the Ph.D. degree from Bhupendra Narayan Mandal University, Madhepura, Bihar, India in 2007. Currently, he is an Assistant Professor of Physics at J.C. Bose University of Science & Technology, YMCA, Faridabad, India. He has published several research papers in field of plasma physics. His research interests are in the area of plasma physics and nonlinear systems.