In quantum plasmas, the Collective interactions between an ensemble of degenerate electrons and positrons/holes give rise to novel waves and structures by Bohm and Pines in 1953 [4, 5]. The basic concept of semiconductor quantum plasma is the de Broglie wavelengths of the plasma particles may be comparable to the Debye length [6] or other scale lengths of the plasma by using magneto hydrodynamic (MHD) model for plasmas, have developed quantum hydrodynamic (QHD) model to study the quantum corrections in plasma characteristics by Haas [7], Manfredi [8] and M. Marklund, P.K. Shukla in 2006 [9]. In quantum plasmas, due to inter-fermion distances much lower than its de Broglie wavelength and the influence of the Pauli exclusion rule, many quantum effects such as electron-tunneling, degeneracy pressure, and Landau quantization may occur.

Quantum plasma shows dispersion instead of dissipation, which is caused by quantum tunneling effects described by Bohm potential term. Whereas dissipation may arise due to kinematic of viscosity, collisions and wave is propagation is governed by interplay between the quantum tunneling and wave particle interactions.

Most of these works have based on quantum hydrodynamic (QHD) model of plasmas. This model is very useful to study the short-scale collective phenomena, such as waves, instabilities, linear, and nonlinear interactions in dense plasmas [10, 12]. By the quantum hydrodynamics model (QHM), the low frequency electrostatic modes have been investigated in weakly coupled quantum plasma. In dusty plasmas, the low frequency dust acoustic waves is strongly correlated in classical plasmas with non-degenerate electrons, ions and charge dust grains have been observed. The nonlinear studies of electrostatic and electromagnetic waves in quantum plasma were based on the generalized quantum hydrodynamical (GQHD) equations [13, 15] for nonrelativistic degenerate electron fluids supplemented by Poisson’s and Maxwell’s equations.

In this paper, the electrons are degenerate and weakly correlated is described by quantum hydrodynamic model (QHM), whereas ions are non-degenerate and strongly correlated is described by generalized hydrodynamic model (GHM).
In this research work, our main emphasis on dispersion relation for nonlinear electrostatic modes in strongly coupled plasmas by using continuity equations and Poisson’s equation. In weakly nonlinear limit, ion-ion correlation effects introduce a viscous dissipation, which is responsible for Korteweg-de Vries Burger equation. The dispersion relation is analyzed theoretically and numerically, and this solution shows existence the shock wave in dissipation dominated.

II. THEORETICAL FORMULATION

In order to study the weakly nonlinear low frequency electrostatic wave propagation characteristics, we take the assumption to solve our problem.

1. The degeneracy parameter for a particle kind $\alpha$ is defined as:

$$\chi_\alpha = \frac{e_{Fe}}{T_\alpha} = \frac{1}{2} \left( \frac{3\pi^2}{\alpha} \right)^{2/3} \left( n_\alpha e_{Fe}^{2/3} \right)^{2/3}$$

where, $e_{Fe} = \hbar^2 \left( \frac{3\pi^2 n_\alpha}{\alpha} \right)^{1/2}$ are the Fermi energy of ground state and $\lambda_{Fe} = \hbar / \sqrt{m_\alpha T_\alpha}$ is the thermal de Broglie wavelength of particle kind $\alpha$.

(i) The electrons are fully degenerate so that the electron Fermi energy ($e_{Fe}$) is much larger than electron thermal energy ($T_e$) and electron degeneracy parameter $\chi_e >> 1$. This shows that

$$n_\alpha >> 1 \left( \frac{2m e T_e^3}{\hbar^2} \right)^{3/2} = n_e Q$$

(ii) The ions are non-degenerate so that ion Fermi energy ($e_{Fe}$) is much smaller than ion thermal energy ($T_i$) and ion degeneracy parameter $\chi_i << 1$. This shows that

$$n_\alpha << 1 \left( \frac{2m_i e T_i^3}{\hbar^2} \right)^{3/2} = n_i Q$$

2. The electron correlations are neglected as electron-electron correlation effects are negligibly small compared to ion-ion correlation. The electron coupling parameter is

$$\Lambda_e = \left( \frac{1}{n_\alpha e_{Fe}^{2/3}} \right)^{2/3} = 1.5Z^{-5/3} \chi_e^{-1} \varphi$$

The electron degeneracy parameter $\chi_e >> 1$, $Z \geq 1$ and $\varphi = (T_i / T_e) \leq 1$. Where $\lambda_{Fe} = v_{Fe} / \sqrt{3} \omega_{pe}$ is the Thomas-Fermi three dimensional screening length of electrons, $\omega_{pe} = \sqrt{\varepsilon_0 e^2 / n_e m_e}$ is the electron plasma frequency and $v_{Fe} = \frac{2e_{Fe}}{m_e}$ is the Fermi speed of electrons. The electron-ion interactions are weak compared to ion-ion correlations and, therefore, we neglect electron-ion interactions.

3. The ions are strongly correlated, i.e., ion coupling parameter is:

$$\Lambda_i = \left( \frac{1}{n_\alpha e_{Fe}^{2/3}} \right)^{2/3} = \frac{Z^2 e^2}{\varepsilon_0 T_i^{3/2}} >> 1$$

where, $e_{Fe} = \sqrt{n_\alpha T_i} / (n_\alpha Z^2 e^2)$ is the ion Debye radius and $\chi_i = (3/4n_\alpha)^{1/3}$ is inter ionic distance. This implies that:

$$n_\alpha >> 3 \left( \frac{Z^2 e^2}{\varepsilon_0 T_i^{3/2}} \right)^{-3} = n_{SC}$$

The assumptions 1 and 3 determine highly dense quantum plasma system $[n_e Q << n_i Q]$. Our assumptions are valid for a typical hydrogen plasma if plasma number density $n_\alpha ~ \left(10^{28} - 10^{32}\right) m^{-3}$ and $\varphi = (T_i / T_e) = 1$. It means physical plasma system is highly dense if the ion coupling parameter $\Lambda_i >> 1$ (strongly coupled) in which electrons with weak interactions form a degenerate quantum fluids whereas ions with strong interactions form a classical fluids.

In order to account for the correlation among ion dynamics, we use viscoelastic approach which is described by general hydrodynamics model [16, 17]. We consider generalized momentum equation for ion fluid using the relation $\frac{d}{dt} = \frac{\partial}{\partial t} + u_i \nabla$ is:

$$\left( 1 + \tau_m \frac{d}{dt} \right) \left( \rho_i \frac{du_i}{dt} + \nabla P_i - Zen_i E \right) =$$

$$= \eta \nabla^2 u_i + \left( \kappa + \frac{\eta}{3} \right) \nabla (\nabla u_i)$$

where, $\tau_m$ is the viscoelastic relaxation time that accounts the memory function, $u_i$ is ion fluid velocity, $\rho = m_i n_i$ is ion mass density, $P_i$ is ion pressure, $\eta$ and $\kappa$ are the shear and bulk coefficients of viscosity.

The general hydrodynamic model also includes ion continuity equation and energy equation. The ion energy equation is not required because ion dynamics is isothermal at strong couplings.

$$\frac{\partial n_i}{\partial t} + \nabla (n_i u_i) = 0$$

(2)

the gradient ion pressure becomes:

$$\nabla P_i = T_i \mu_i \nabla n_i$$

where, $\mu_i$ is the coefficient of isothermal compressibility for ion fluid.

The conservation of momentum equation for electron is:

$$0 = -e E + \frac{\nabla P_e}{n_e} + \frac{\hbar^2}{2m_e} \nabla \left( \frac{\nabla^2 n_e}{\sqrt{n_e}} \right)$$

(3)

where, $n_e$ is unperturbed electron number density, $P_e$ is electron pressure and the term $\hbar^2$ arises due to electron tunneling through the Bohm potential [18]. The system of equations is closed by Poisson’s equation.
\[ \nabla E = \frac{e}{\varepsilon_0} (Zn_i - n_e) \]  

(4)

In strongly coupled quantum plasma, we considered the ion viscous dissipation effects the weakly nonlinear structures in 1-dimensional in the hydrodynamic range \((\omega \tau_0 \ll 1)\).

To explore the nonlinear structures, it is convenient to write governing equations in dimensionless form. We use following dimensionless variables:

\[ \frac{\partial u_i}{\partial \tau} + u_i \frac{\partial u_i}{\partial x} + \mu_i \frac{\partial \ln n_i}{\partial x} - \bar{E} = \frac{\eta^*}{n_i} \frac{\partial^2 u_i}{\partial \delta^2} \]  

(5)

\[ \frac{\partial n_i}{\partial \tau} + \frac{\partial (n_i u_i)}{\partial x} = 0 \]  

(6)

\[ \frac{\partial E}{\partial x} = n_i - n_e \]  

(7)

where, \( \bar{H} = \frac{H}{3} = \frac{h\omega_{pe}}{2T_{Fe}} \) the quantum diffusion due to 1-dimensional electron tunneling effect and \( \lambda_{Fe} = \sqrt{3} \lambda_{De} = v_{Fe}/\omega_{pe} \) is the Thomas-Fermi 1-dimensional screening length of electrons.

Now, we derive the Korteweg-de Vries equation from (5)-(7) by employing the reductive perturbation technique and the stretched coordinates:

\[ \delta = e^{\nu_x/2} (x - Mt) \]  

(8)

\[ \tau = e^{\nu_x/2} t \]  

(9)

where, \( \varepsilon \) is a smallness parameter proportional to the amplitude of the perturbation and \( M \) is the mode normalized by the ion thermal speed.

We can expand the variables \( n_{e(i)}, u_i, E \) in a power series of \( \varepsilon \) as:

\[ n_{e(i)} = 1 + \varepsilon n_{e(i)}(1) + \varepsilon^2 n_{e(i)}(2) + \ldots \]  

(10)

\[ u_i = 0 + \varepsilon u_i(1) + \varepsilon^2 u_i(2) + \ldots \]  

(11)

\[ E = 0 + \varepsilon^{3/2} E(1) + \varepsilon^2 E(2) + \ldots \]  

(12)

Now, using (10)-(12) in (5)-(8) and taking the coefficient of \( e^{3/2} \) from (12) and \( \varepsilon \) from (10)-(11), we get:

\[ E_1 = \frac{\partial}{\partial \delta}\left( \mu n_{e(i)}(1) - Mu_{i(1)}(1) \right) \]  

(13)

\[ u_{i(1)} = Mn_{i(1)} \]  

(14)

\[ E_{1(1)} = -\left( \frac{\lambda_{Fe}}{\lambda_{De}} \right)^2 \frac{\partial n_{e(i)}(1)}{\partial \delta} \]  

(15)

\[ n_{e(1)} = n_{e(1)}(1) \]  

(16)

Now, using (13)-(16) we have:

\[ M = \left( \mu + \left( \frac{\lambda_{Fe}}{\lambda_{De}} \right)^2 \right)^2 \]  

(17)

Now substituting (13)-(16) into (5)-(7) and equating the coefficient of from (10)-(12), we obtain:

\[ E^{(2)} = \frac{\partial u_{i(1)}}{\partial \tau} + u_{i(1)} \frac{\partial u_{i(1)}}{\partial \delta} - \mu n_{i(1)} \frac{\partial n_{i(1)}}{\partial \delta} - \eta \frac{\partial^2 u_{i(1)}}{\partial \delta^2} + \mu \frac{\partial n_{i(2)}}{\partial \delta} - M \frac{\partial u_{i(2)}}{\partial \delta} \]  

(18)

\[ \frac{\partial n_{i(1)}}{\partial \tau} + \frac{\partial (n_{i(1)} u_{i(1)})}{\partial \delta} = -M \frac{\partial u_{i(2)}}{\partial \delta} \]  

(19)

\[ E^{(2)} = \left( \frac{\lambda_{Fe}}{\lambda_{De}} \right)^2 \frac{\partial n_{e(i)}(1)}{\partial \delta} - \left( \frac{\lambda_{Fe}}{\lambda_{De}} \right)^2 \frac{\partial n_{e(i)}(2)}{\partial \delta} + \frac{\bar{H}^2}{4} \left( \frac{\lambda_{Fe}}{\lambda_{De}} \right)^4 \frac{\partial^3 n_{e(i)}(1)}{\partial \delta^3} \]  

(20)

\[ \frac{\partial E_{1(1)}}{\partial \delta} = n_{e(2)} - n_{e(2)} \]  

(21)

Now, using above equation and eliminating \( n_{e(i)}, u_i \) and \( E \), we obtain:

\[ \psi = \frac{M n_{i(1)}^{(1)}}{\mu_2 + \left( \frac{\lambda_{Fe}}{\lambda_{De}} \right)^2} \]  

(22)

\[ \frac{\partial \psi}{\partial \tau} + \eta \frac{\partial \psi}{\partial \delta} + \lambda^3 \frac{\partial^3 \psi}{\partial \delta^3} = -\frac{2 \mu}{\mu_2} \]  

(23)

where, the coefficients \( A \) and \( \mu \) are given by

\[ A = \frac{1}{2M} \left( \frac{\lambda_{Fe}}{\lambda_{De}} \right)^2 \left( 1 - \frac{\bar{H}^2}{4} \right) \]  

(24)

\[ \mu = \frac{\eta^*}{2} \]  

(25)

Equation (23) is the Korteweg-de Vries Burger equation of the weakly nonlinear low frequency electrostatic wave in strongly coupled quantum plasma. The solution of Korteweg-de Vries equation is found by transforming the independent variables \( \delta \) and \( \tau \) to:

\[ K = \delta - C_0 \tau, \tau = \tau \]  

where, \( C_0 \) is a constant velocity normalized by \( c \).

III. NUMERICAL SOLUTION AND DISCUSSION

In order to get the shock structure, it is necessary to apply the boundary condition on wave. The exact solution of KDVB is not possible because this equation is not exactly integral solution. A particular solution of KDVB is possible, which are only for monotonic shock structure. Actually, a monotonic shock structure is possible only when dissipation dominates, and oscillatory shock structure is possible only when dissipation is weak. The boundary condition is:

\[ \psi \rightarrow 0, \quad \frac{d\psi}{dK} \rightarrow 0, \quad \frac{d^2\psi}{dK^2} \rightarrow 0 \text{ at } K \rightarrow \infty \]  

Finally, Equation (22) becomes:

\[ \frac{d^2 \psi}{dK^2} = \left( \frac{C_0}{A} \right) \psi - \left( \frac{1}{2A} \right) \psi^2 + \left( \frac{\mu}{A} \right) \frac{d\psi}{dK} \]  

(26)
Equation (26) is well known as the damped harmonic oscillator in which $\psi$ describes the generalized coordinate and $\delta$ describes the time. Equation (26) has two singular points:

(i) $\psi \to 0$, $\frac{d\psi}{dK} \to 0$

(ii) $\psi \to 2C_0$, $\frac{d\psi}{dK} \to 0$

The first point shows the equilibrium downstream state and the second point shows the upstream state. If we assume, $K = \infty (\delta = \infty)$, the particle was located at $\psi = 0$ and if $K = -\infty (\delta = -\infty)$, the particle at point $\psi = 2C_0$.

The shock strength is:

$$Shock\ strength = \psi(-\infty) - \psi(+\infty) = 2C_0$$

The Mach number is independent of dispersion:

$$MA = \frac{Nonlinear\ Wave\ Velocity}{Linear\ Wave\ Velocity}$$

$$MA = 1 + e \left( \frac{C_0}{M} \right)$$ (27)

To get the nature of shock structure, the solution of Equation (26) is obtained by substitute the $\psi = 2C_0 + \varphi$, where $2C_0 >> \varphi$.

$$\frac{d^2\varphi}{dK^2} + \left( \frac{C_0}{A} \right) \varphi - \left( \frac{\mu}{A} \right) \frac{d\varphi}{dK} = 0$$ (28)

The solution of equation (28) is proportional to $\exp(\sigma K)$, where:

$$\sigma = \frac{\mu}{2A} \pm \sqrt{\frac{\mu^2 - C_0}{4A^2}}$$ (29)

$\bar{H} = 0.2$

The oscillatory shock structure in which dispersion dominates over dissipation by different values of $\tau$ as shown in Figure 1 and this follows the Equation (29) with the singular point $(2C_0, 0)$. It is clear from Figure 1(d), the oscillatory shock is fully developed at $\tau = 1200$ with singular point $2C_0 = 0.1$ giving shock speed $C_0 = 0.05$, dispersion coefficient $A = 3$ and burger coefficient $\mu = 10^{-2}$.

$$\psi = C_0 \left[ 1 - \tanh \left( \frac{C_0 K}{2\mu} \right) \right]$$ (30)

$\bar{H} = 2$

The monotonic shock structure in which dissipation dominates over dispersion by different values of $\tau$ as shown in Figure 2 and this follows the Equation (30) with the singular point $(2C_0, 0)$. In Figure 2, the dispersion coefficient $A = 0$, $C_0$ is the amplitude and $2\mu/C_0$ is width of the shock and other values are same as in Figure 1. In this range, burger coefficient $\mu = 10^{-2}$, dispersion coefficient $A = 1 \sim 3$ and $0 < H < 2$ [19, 20], monotonic shock structure is well agree with Equation (30).
Figure 1(d). Oscillatory shock structure at $\tau=1200$

Figure 2(a). Monotonic shock structure at $\tau=0$

Figure 2(b). Monotonic shock structure at $\tau=300$

Figure 2(c). Monotonic shock structure at $\tau=500$

Figure 2(d). Monotonic shock structure at $\tau=800$

IV. CONCLUSION

The propagation of nonlinear low frequency electrostatic modes in strongly coupled quantum plasma has investigated. The behavior of strongly coupled quantum plasma is the collective nature, which is most important property of this plasma. This investigation supports the existence of shock wave due to ion-ion correlation in high energy density of strongly coupled quantum plasma. The oscillatory shock structure in which dispersion dominates over dissipation and monotonic shock structure in which dissipation dominates over dispersion by different values of $\tau$ are discussed. The results may be significant for understanding the scattering process involving intense laser beam in high energy density compressed plasma experimentally. In dissipative plasma, the propagation of small but finite amplitude nonlinear excitations maybe described by Korteweg-de Vries Burger equation.
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 BIOGRAPHIES

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