

DATA-DRIVEN FINITE HORIZON CONTROL BASED ON GAUSSIAN PROCESSES AND ITS APPLICATION TO BUILDING CLIMATE CONTROL

Sh. Abdufattokhov^{1,*} K. Ibragimova² M. Khaydarova² A. Abdurakhmanov²

1. Control and Computer Engineering Department, Turin Polytechnic University in Tashkent, Tashkent, Uzbekistan, shokhjakhon2010@gmail.com

2. Computer Science Department, Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi, Tashkent, Uzbekistan, komila.ibragimova@inbox.ru, khaydarovamarkhmat@gmail.com, abba1963@mail.ru

**. Corresponding Author*

Abstract- Because of a stable increase in energy usage for thermal comforts in buildings, the role of sustainable energy effectiveness in the building sector has become one of the concerning issues since the last few decades to overcome environmental impacts of energy-producing plants to be ecologic friendly with nature. Incorporating machine learning methods to set up a reliable control scheme seems to be a promising idea to handle the energy efficiency of complex building dynamic systems with uncertainties, while data-driven control laws are becoming well-off in the system engineering community. In this paper, a data-driven approach is considered to choose energy set points to manage climate control up to satisfactory level through incorporating Gaussian Processes method, where the predictive model is built by correlating the dynamics, complexity and interrelated energy consumption recordings of the building into Finite Horizon Control (FHC) technique that provides (sub)optimal solutions for each time step. In the end, a demonstrative example is illustrated to give a clear picture of the proposed modelling method and its potentials in the case of building climate control.

Keywords: Gaussian Processes, Data-Driven Control, Uncertain Dynamic Systems, Climate Control.

1. INTRODUCTION

Improvements in the distribution of total energy economically optimal are one of the major prerequisites to fulfil facility demands in building areas due to high and fluctuating prices in local and global energy markets. Building facility managers society still lacks novel ideas to overcome energy efficiency concerns, although recent innovative approaches have been discovered and being implemented consistently in utilizing the energy management requirements fully. From the ecological perspective, the role and contribution of continuous reductions in energy-consuming to cut off GHG emission impact are crucial in jumping towards an eco-friendly environment. Furthermore, the traditional modelling approach is not the most effective because it can consider

system dynamics and the dynamic effect of system disturbances/noises. For these reasons, identifying energy-related problems have become a hot area of interest in recent years among scientists from different areas. Hesselbach, et al. [1] and Herrmann et al [2] proposed their state of the art for optimized process chains and locations of technical building services. Devoldere, et al. [3],[4] carried out their research on the energy-related impact and cost reduction proposals for machine design in a production line. The combinations of power metering with sensors to monitor energy management systems was another considerable work by authors of [5-7]. On the other hand, Zile [8] tested the performance of artificial neural network-based data-driven control idea and showed the proposed technique has a promising future. Because modelling a building's energy system with a conventional method is time-consuming, many engineers from different fields get an accurate model for satisfactory prediction, and alternative approaches are needed.

In this paper, our main contribution is to investigate the problem, as mentioned earlier, through discussions on how Artificial Intelligence (AI) can be applied to data collected from sensors located in buildings. Specifically, we implement the Gaussian Processes (GP) technique [9] that enables us to get a predictive model by incorporating the propagation of the disturbance uncertainty into the system states. In order to achieve energy-efficient management using Finite Horizon Control (FHC), alternatively called Model Predictive Control, that has been applied to real systems and showed to be an efficient supervisory control solution providing energy savings with better thermal comfort over rule-based control [10],[11], where a statistical model provides the estimation of the future response of a plant. By combining the algorithms mentioned above, one can design a reliable controller that reduces both cost and time as we directly work with the data without explicitly modelling internal states. The proposed method is scalable to identify a control-oriented model repeatedly given the historical data from the building.

The remainder of the paper content is organized as follows: In section 2 main energy consumers are introduced and an approach on how data can be collected is presented. Next two sections 3 and 4 are devoted for the methodology of predictive modeling using GP and its application to finite horizon control, respectively. In section 5, a demonstrative example is given and advantages of the proposed method is shown. Finally, we draw up conclusions and glance at possible future works in section 6.

2. DATA COLLECTION

In general, total energy supplied to building sectors such as offices, universities, multi-storey houses and etc. wasted for auxiliary services that includes chillers, air-compressors, boilers and lighting, in general mainly HVAC (Heating and Ventilation of Air Conditioning) systems. Besides, there exist three primary energy emissive sources: heat transferred from the ambient environment by radiation from sunshine, and the last one, heat coming from doors or windows openings. The Figure 1 illustrates the possible factors affecting energy distribution from different perspectives, and one can see the relationships are complicated, non-linear, dynamic and uncertain. Although it may seem possible to model their dynamic correlations theoretically with acceptable accuracy for a realistic understanding of their behavior from a physical engineer's point of view, in reality, control their performance for energy efficiency is extremely difficult.

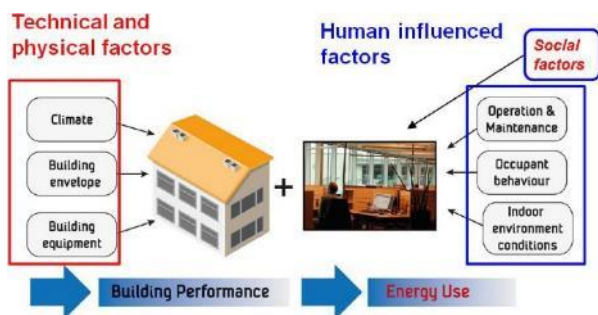


Figure 1. Factors affecting to the energy consumption in buildings [12]

One way to overcome difficulties arising in traditional modelling methodologies is to rely on data-driven modelling that collects energy consumption and operation data, and develop a system model. The data-driven approach decreases both cost and time significantly if one works with the sensor data without explicitly modelling internal states. Another advantage of trusting data-driven system identification is that the method is scalable to identify a control-oriented model for a different building given the historical data.

Since our focus is modeling energy system with uncertainties, the power consumption and thermal comfortless are two main issues to be handled in building energy management. We define \mathbf{p} is an objective parameter (i.e. power, energy, temperature measurements) and it is defined by several output measurements \mathbf{y} , which

is mostly formulated as a function of control inputs \mathbf{u} in dynamical modeling. Measurements of the systems can be obtained through either SCADA (supervisory control and data acquisition) software system or directly from relevant digital sensors as time series data. We can collect time series data matrix \mathbf{M}^a as follows

$$\mathbf{M}^a = [\mathbf{p}^a \ \mathbf{y}^a \ \mathbf{u}^a] = \begin{cases} \mathbf{p}^a = [p_i^a], i=1,2,\dots,N \\ \mathbf{y}^a = [y_{ij}^a], i=1,2,\dots,N; j=1,2,\dots,N_p \\ \mathbf{u}^a = [u_{ik}^a], i=1,2,\dots,N; k=1,2,\dots,N_u \end{cases} \quad (1)$$

where, a is superscript for a type of energy consuming equipment; i is time interval, j th output and k th input parameter subscripts, respectively; N is the total number of the collected data; N_p is the total number of output parameters; N_u is the total number of input parameters. For example, u_{ik}^a stands for the value of input parameter k of machine a at index i .

3. LEARNING UNCERTAIN ENERGY SYSTEM MODEL USING GAUSSIAN PROCESSES

In most deterministic machine learning algorithms, difficulties in the training process stem from a lack of inefficient data. When a model is chosen, examining directions anticipated from this model leave the training data. Although the capacity approximator's forecasts are discretionary, they are guaranteed with "full certainty" [13]. To overcome the issue, building up a model dependent on appropriate intelligent algorithms which fabricates the framework's model utilizing stochastic capacity approximator that puts a back dispersion over the mapping capacity and communicates the degree of vulnerability about the model [14] can be another option and practical arrangement. Thus, to model the system without any prior knowledge, we initially require a probabilistic model to cover model uncertainties. For this purpose, we can use non-parametric probabilistic Gaussian Processes to design a stochastic model. In order to visualize what is aimed to construct in the paper, refer to Figure 2.

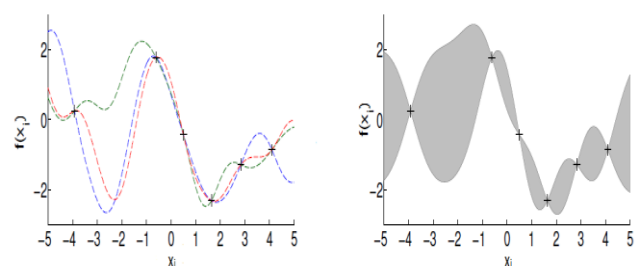


Figure 2. '+' - training samples. Deterministic function approximators (left) and Probabilistic function approximator (right) [11]

3.1. Gaussian Processes

A Gaussian Processes (GP) is a batch of random variables, which form Gaussian distribution jointly. The GP models is a member of a nonparametric class methods for nonlinear system identification that computes a new prediction of system through incorporating Bayesian

inference techniques applied to historical data sets [15]. GP models have been successfully implemented in different research fields [16-19], due to possibility to include various kinds of prior knowledge into the model [20], [21] for the incorporation of local models and the static characteristic that makes the method more attractive as compared to other regression algorithms.

A GP is completely specified by its mean function and covariance function. It is very common to define mean function $m_f(x)$ and the covariance function $C_f(x_i, x_j)$ for pair (i, j) of a dynamic process $f(x)$ under consideration as

$$m_f(x_i) = E[f(x_i)] \tag{2}$$

$$C_f(x_i, x_j) = E[(f(x_i) - m_f(x_i))(f(x_j) - m_f(x_j))] \tag{3}$$

In order to develop a prognostic model using predefined data in Section 2, we use GPs, please refer to [13] for more brief details.

Consider the system

$$y = f(x) + \epsilon$$

with the white Gaussian noise $\epsilon \sim N(0, \sigma_n^2)$, with the variance σ_n^2 and the vector of regressors x from the input

dimension space R^D that includes autoregressive terms of outputs and inputs of the building energy system, and y is corresponding output measurement with respect to input vector x . We assume that

$$[y_1, \dots, y_N]^T \sim GP(0, K)$$

with $K = K_f + \sigma_n^2 I$ where $K \in R^{N \times N}$, $K_f \in R^{N \times N}$ is the covariance matrix for the noise-free system computed by using the covariance function $C_f(x_i, x_j)$ that is applied to all the pairs i and j of the training data D , I is the $N \times N$ identity matrix, and T stands for a transpose of a matrix.

We refer to [21] for a reader to access more information about mean and covariance functions together with its implementation in GP models. Below, we give an example of the composite covariance function composed of the squared exponential covariance function and the constant covariance function because of uncertainties caused by environment

$$C(x_i, x_j) = \sigma_f^2 \exp \left[-\frac{1}{2} \sum_{d=1}^D \theta_d (x_i^d - x_j^d)^2 \right] + \sigma_n^2 \delta_{ij} \tag{4}$$

3.2. Prediction with GP

To forecast a new output y^* of the GP model for a given input x^* , we rely on the Bayesian framework. For the batch of random variables $[y_1, \dots, y_N, y^*]$ we define

$$Y_{N+1} \sim GP(0, K_{N+1}).$$

with the covariance matrix

$$K_{N+1} = \begin{pmatrix} K & K_* \\ K_*^T & K_{**} \end{pmatrix}$$

where, $K_{N+1} \in R^{N+1 \times N+1}$, $K_* \in R$, $K_{**} \in R$ with

$K_* = [C(x_1, x^*), \dots, C(x_N, x^*)]^T$ is the $N \times 1$ vector of covariances between the training samples and the test input sample, $K_{**} = C(x^*, x^*)$ is the autocovariance submatrix of the test input data.

Finally, we end up with the Gaussian prediction with the following mean and variance

$$E[y^*] = \mu(x^*) = m_f(x^*) + K_*^T K^{-1} (Y - m_f(X)) \tag{5}$$

$$\text{var}[y^*] = \sigma^2(x^*) = K_{**} - K_*^T K^{-1} K_* \tag{6}$$

4. DATA-DRIVEN FINITE HORIZON CONTROL BASED ON GP

In this section, we first give an introduction to FHC and talk about its components. Afterwards, we incorporate GP to FHC and form data-driven control.

4.1. Introduction to FHC and its Components

Finite Horizon Control (FHC) is widely used control algorithm that predicts the future response of a plant using an explicit process model for an industrial use. The prediction horizon N_h range together with optimization ability of FHC algorithms to handle with constraints, that are often met in control practice, have made it popular and widely used compared to other approaches in many applications [22-26]. The FHC working standard is demonstrated in Figure 3 adopted from [27] and is summed up as follows:

1. Prediction of framework yield signal $y(i+h)$ is determined for each discrete time instant i using a finite prediction horizon $h = 1, \dots, N_h$. Predictions are labeled as $\hat{y}(i+h|i)$ where h is a step ahead estimation from time i , while N_h and N_c is an upper bound for prediction and control horizons, accordingly.
2. The vector of optimal control signal $u(i+h|i)$ for $h = 0, \dots, N_h - 1$ is determined by minimization of estimation error $\hat{y}(i+h|i)$ for $h = 1, \dots, N_h$.
3. Only the first component of the optimal control signal vector is applied. In the following steps, another deliberate yield test is repeated and the entire procedure described above is circled inside the loop.

4.2. FHC Based on GP

Combining input-output model of dynamic system with our GP model, we write our dynamical system as follows

$$p(i) = f(x(i)) + \epsilon(i) \tag{7}$$

where,

$$x(i) = [p(i-l_1), \dots, p(i-1), u(i-l_2), \dots, u(i), d(i-l_3), \dots, d(i)] \tag{8}$$

with $f \sim GP(\mu_f, \sigma_f^2)$, ϵ is measurement noise, p is the (past) output, u is the control input, d is the exogenous disturbance input and l_1, l_2, l_3 are the lags for autoregressive outputs, control inputs, and disturbances, respectively. The autoregressive lag l_1 denotes the model order and selected manually, but mostly less than 5 in practice.

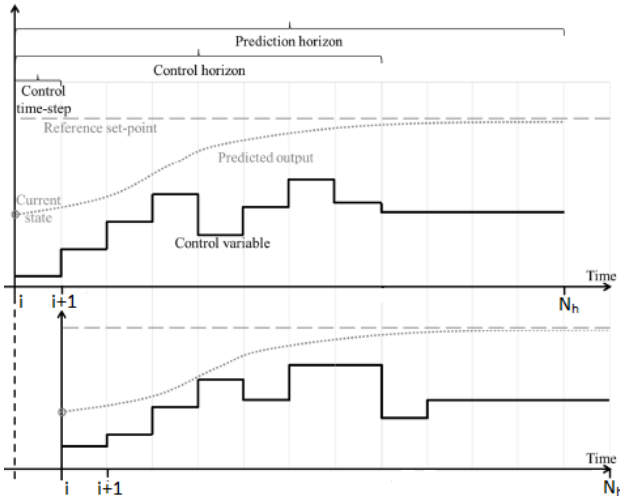


Figure 3. Schematic of the principle of finite horizon control, the difference between the top and the bottom subplots is one time step [27]

Now let's focus on our FHC optimization problem. Thanks to the availability of uncertainty prediction in GP modelling, including the uncertainty term into FHC optimization objective enables to design a robust controller that will optimize action according to the validity of model. Overall, the optimization problem with quadratic cost is

$$\begin{aligned} & \text{minimize} \quad \sum_{h=0}^{N_h-1} \|\hat{p}^s(i+h)\|_Q^2 + \hat{\sigma}^{s^2}(i+h) + \|u^s(i+h)\|_R^2 \\ & \text{subject to} \quad \hat{p}^s(i+h) = m_f^s(x^s(i+h)) + K_*^s K^{s-1} (Y^s - m_f^s(X^s)) \\ & \quad \hat{\sigma}_s^2(i+h) = K_{**}^s - K_*^s K^{s-1} K_*^{sT} \quad (9) \\ & \quad \hat{p}^s(i+h) \in P^s \\ & \quad u^s(i+h) \in U^s, \quad h = 1, \dots, N_h \end{aligned}$$

where, s stands for a type of energy consuming devices a, b, c, \dots in a building; P is state output constraint set, U is set of feasible solutions; $\|x\|_A^2 = x^T A x$ Euclidian norm for $x \in R^n$ and Q, R are positive definite matrices.

5. A NUMERICAL EXAMPLE

5.1. System Description

To test the proposed data-driven FHC based on GP, we examine a simplified version of the building example

given in [28]. The accompanying discrete model below in Equation (9) outlines the utilization of proposed GP strategy for system identification of highly fluctuating and non-periodic system. Once the appropriate GP model is selected, we incorporate the model into FHC problem. All simulations were carried out in Matlab software and Intel Core i5-5200U.

Consider the following discrete nonlinear system

$$\begin{cases} y(i+1) = Ay(i) + Bu(i) + Ev(i) \\ p(i) = Cy(i) + w(i) \end{cases} \quad (10)$$

$$\text{with } A = \begin{bmatrix} 0.8511 & 0.0541 & 0.0707 \\ 0.1293 & 0.8635 & 0.0055 \\ 0.0989 & 0.0032 & 0.7541 \end{bmatrix}, \quad B = \begin{bmatrix} 0.070 \\ 0.006 \\ 0.004 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.02221 & 0.00018 & 0.0035 \\ 0.00153 & 0.00007 & 0.0003 \\ 0.10318 & 0.00001 & 0.0002 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T$$

Here, we denote the outputs that consists of the temperature in the wall connected with another room, the room temperature and the temperature in the wall connected to the outside. The system was subject to three disturbances, namely the outside temperature, the solar radiation and the internal heat gains (e.g., number of people, electronic devices and etc). The objective of the given model is (room temperature) that is disrupted with Gaussian white noise with (assumed as weather disturbance), and should be kept above with minimum energy by controlling the heating .

5.2. Modeling with GP

To obtain a model for given inputs of the discrete-time system described by Equation (9) based on statistical data and by following step by step the proposed methodology in the section 3, we generate control input of 400 by keeping signals consecutive 4-time instants (4 hours) with uniform distribution in the magnitude between 0 and 45, and collect the corresponding output measurements of the system. We use 50 of the generated data set (the rest is used for testing). We validate models with metrics *NRMSE* (Normalizes Root Mean Square Error) and *MSLL* (Mean Standard Log Loss) provided in [21]. The bigger the former is, the better the accuracy is, while this holds vice versa for the latter. In practice, GP models with zero mean is common, so we simply set 0 and seek proper covariance function. Models of various orders (autoregressive lags) together with several composite covariance functions are fitted in GP model to select the best covariance function. As a result, the composition of sum of squared exponential and periodic covariance functions is chosen together with $l_1 = 2$, $l_2 = 1$, and $l_3 = 0$ as it performs better accuracy, as shown in Table 1 and Figure 4.

Moreover, we test the model with different Gaussian noises in Figure 5, one can see that, even though test data fitting graph has larger variance, it still captures the trajectory well.

Table 1. GP modeling accuracy results for test data using different model order and covariance functions (se - squared exponential, se+per - periodic, rq - rational quadratic).

Model order	NRMSE			MSLL		
	se	se+per	rq	se	se+per	rq
$l_1 = 3,$ $l_2 = 2,$ $l_3 = 1$	0.051	0.002	0.094	-1.874	-2.131	-1.029
$l_1 = 2,$ $l_2 = 1,$ $l_3 = 0$	0.095	0.001	0.045	-0.987	-3.285	-2.009
$l_1 = 2,$ $l_2 = 1,$ $l_3 = 1$	0.017	0.043	0.096	-1.771	-2.946	-2.092
$l_1 = 1,$ $l_2 = 0,$ $l_3 = 0$	0.115	0.008	0.067	-1.284	-2.846	-1.088

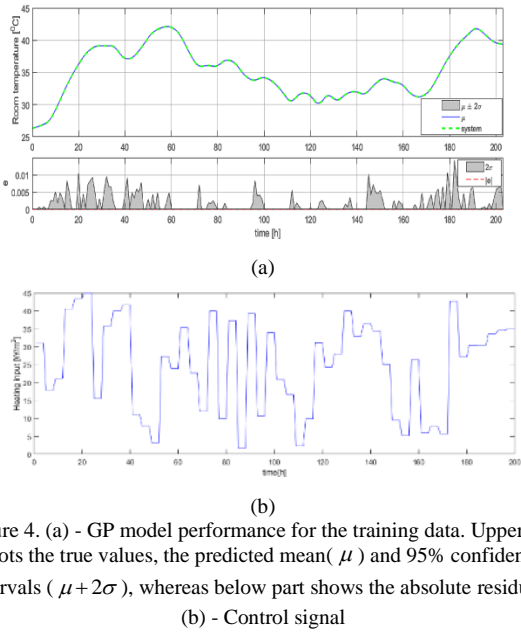


Figure 4. (a) - GP model performance for the training data. Upper part plots the true values, the predicted mean (μ) and 95% confidence intervals ($\mu + 2\sigma$), whereas below part shows the absolute residuals. (b) - Control signal

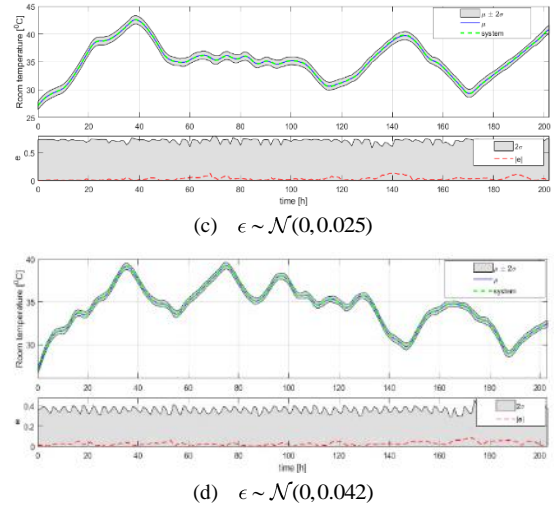
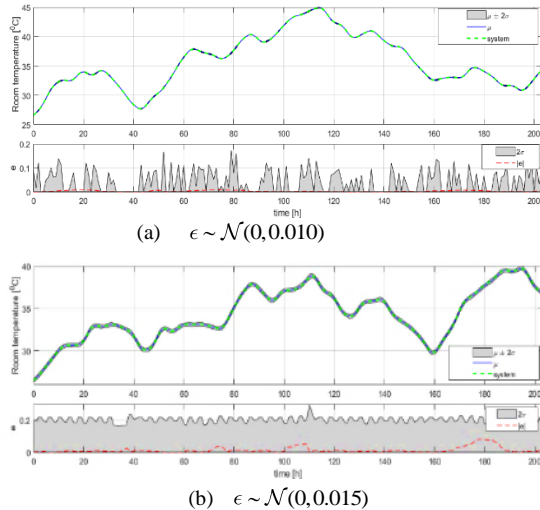


Figure 5. Model performance with different Gaussian noises

5.3. Data-driven FHC based on GP

To show the potentials of the proposed approach, initially we solve the following FHC optimization problem with $Q=1$ and $R=0.5$

$$\begin{aligned}
 & \text{minimize} && \sum_{h=0}^6 \|p(i+h)\|_Q^2 + \|u(i+h+1)\|_R^2 \\
 & \text{subject to} && y(i+h+1) = Ay(i+h) + Bu(i+h) + Ev(i+h) \\
 & && p(i+h) = Cy(i+h) + w(i+h) \quad (11) \\
 & && 21 [^0C] \leq p(i+h) \\
 & && 0 [W/m^2] \leq u(i+h) \leq 45 [W/m^2] \\
 & && h = 1, \dots, 6
 \end{aligned}$$

Afterwards, to compare the results with the traditional approach above, we solve the GP based FHC problem below with the same Q and R

$$\begin{aligned}
 & \text{minimize} && \sum_{h=0}^6 \|\hat{p}(i+h)\|_Q^2 + \hat{\sigma}^2(i+h) + \|\hat{u}(i+h)\|_R^2 \\
 & \text{subject to} && \hat{p}(i+h) = m_f(x(i+h)) + K_* K(Y - m_f(X)) \\
 & && \hat{\sigma}^2(i+h) = K_{**} - K_* K^{-1} K_*^T \quad (12) \\
 & && 21 [^0C] \leq \hat{p}(i+h) \\
 & && 0 [W/m^2] \leq \hat{u}(i+h) \leq 45 [W/m^2] \\
 & && h = 1, \dots, 6
 \end{aligned}$$

where, we choose the input parameter vector for the GP model as $x(i+h) = [\hat{p}(i-2+h) \hat{p}(i-1+h) \hat{u}(i-1+h) \hat{u}(i+h) w(i+h)]$ for $h = 1, \dots, 6$.

During simulations, we used IPOPT algorithm as NLP solver in the CasADi framework [29]. Moreover, we take a PID control [30] method which is widely and frequently implemented industrial controller as baseline for a fair comparison. From the results, we observe that FHC via GP law in Equation (12) can be expected to outperform PID control technique, whereas with the classic FHC formulation in Equation (11) almost a similar performance is achieved. We refer Figures 6 and 7 for room temperature and control results, respectively.

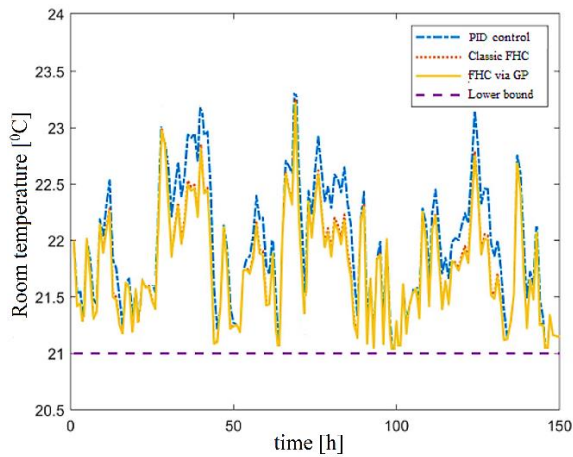


Figure 6. Room temperature comparison of different control methods

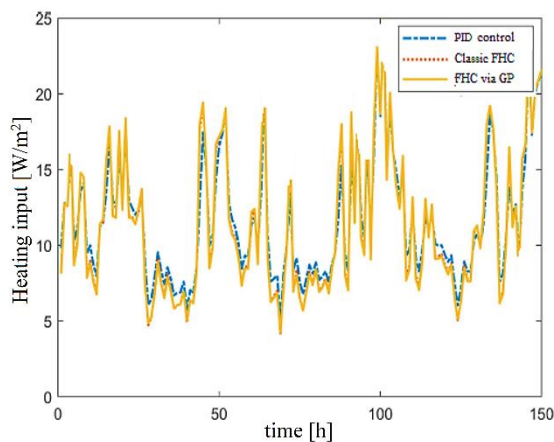


Figure 7. Optimal heating set point values of different control methods

6. CONCLUSIONS

In this paper, we tried to show the possibility to model a dynamic system using machine learning. Specifically, Gaussian Processes applied to historical data collected from sensors of a building. Once we have defined the data collection method and modelling the building's uncertain energy system with GP, we implemented the method to the climate control system in an optimized way, where optimal solutions are defined by the FHC loop each control time step. In the end, the numerical example demonstrates to give a picture of GP modelling potentials. The proposed approach can be accepted as a new tool for identifying a building's complex dynamic system relying on GP and quantification of their respective energy saving perspectives while guaranteeing thermal comfort. Our future work will focus on the interpretability and advantages of the proposed method through experimental results based on real system dynamics data.

ACKNOWLEDGEMENTS

Authors are grateful from Turin Polytechnic University in Tashkent, Uzbekistan for providing with necessary software and hardware in order to make simulations.

REFERENCES

- [1] J. Hesselbach, C. Herrmann, R. Detzer, L. Martin, S. Thied, B. Ludemann, "Energy Efficiency through Optimized Coordination of Production and Technical Building Services", 15th CIRP International Conference on Life Cycle Engineering, 17-19 March, 2008.
- [2] C. Herrmann, S. Thiede, "Process Chain Simulation to Foster Energy Efficiency in Manufacturing", CIRP Journal of Manufacturing Science and Technology, Vol. 1, No. 4, pp. 221-229, 2009.
- [3] T. Devoldere W. Dewulf, W. Deprez, B. Willems, JR. Dufloy, "Improvement Potential for Energy Consumption in Discrete Part Production Machines", 15th CIRP International Conference on Life Cycle Engineering, 2007.
- [4] T. Devoldere W. Dewulf, W. Deprez, "Energy Related Life Cycle Impact and Cost Reduction Opportunities in Machine Design: The Laser Cutting Case", 15th CIRP International Conference on Life Cycle Engineering, 2008.
- [5] C. Herrmann, G. Bogdanski, A. Zein, "Industrial Smart Metering - Application of Information Technology Systems to Improve Energy Efficiency in Manufacturing", 43rd CIRP International Conference on Manufacturing Systems, 2010.
- [6] A.M. Hashimov E.E. Novruzova, "New Technologies In Energy Sector and Automated Energy Accounting Systems and their Main Factors of Influence on Ecology", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 42, Vol. 12, No. 1, pp. 53-57, March 2020.
- [7] C. Bauerdick, M. Helfert, B. Menz, E. Abele, "A Common Software Framework for Energy Data Based Monitoring and Controlling for Machine Power Peak Reduction and Workpiece Quality Improvements", Procedia CIRP, Vol. 61, pp. 359-364, 2017.
- [8] M. Zile, "Improved Control of Transformer Centers Using Artificial Neural Networks", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 40, Vol. 11, No. 3, pp. 23-33, September 2019.
- [9] C.K.I. Williams, C.E. Rasmussen, "Gaussian Processes for Regression", Advances in Neural Information Processing Systems, Vol. 8, pp. 514-520, MIT Press, 1996.
- [10] D. Sturzenegger, D. Gyalistras, M. Morari, R. Smith. "Model Predictive Climate Control of a Swiss Office Building: Implementation, Results, and Cost-Benefit Analysis", IEEE Transactions on Control Systems Technology, Vol. 24, No. 1, pp. 1-12, 2016.
- [11] Sh. Abdufattokhov, K. Ibragimova, D. Gulyamova, K. Tulaganov, "Gaussian Processes Regression based Energy System Identification of Manufacturing Process for Model Predictive Control", International Journal of Emerging Trends in Engineering Research, Vol. 8, No. 9, pp. 4927-4932, 2020.
- [12] H. Yoshino, T. Hong, N. Nord, "IEA EBC Annex 53: Total Energy Use in Buildings - Analysis and Evaluation Methods", Energy and Buildings, Vol. 152, pp. 124-136, 2017.
- [13] Sh. Abdufattokhov, B. Muhiddinov, "Probabilistic Approach for System Identification using Machine

Learning", International Conference on Information Science and Communications Technologies (ICISCT), IEEE Proceedings, 2019.

[14] M.H. Aliabadi, N.M. Tabatabaei, S.R. Mortezaei, "Energy Storage System and CVAR Minimization in Microgrid Operation under Uncertainties Considering", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 39, Vol. 11, No. 2, pp. 6-12, June 2019.

[15] K. Thompson, "Implementation of Gaussian Process Models for Non-Linear System Identification", Ph.D. Thesis, University of Glasgow, Glasgow, 2009.

[16] J.Q. Candela, C.E. Rasmussen, "Analysis of Some Methods for Reduced Rank Gaussian Process Regression", Lecture Notes in Computer Science, Vol. 1, pp. 33-55, 2005.

[17] K. Azman, J. Kocijan, "Application of Gaussian Processes for Black-Box Modelling of Biosystems", ISA Transactions, Vol. 46, No. 4, pp. 443-457, 2007.

[18] G. Gregorcic, G. Lightbody, "Nonlinear System Identification: From Multiple-Model Networks to Gaussian Processes", Engineering Applications of Artificial Intelligence, Vol. 21, pp. 1035-1055, 2008.

[19] Sh. Abdufattokhov, B. Muhiddinov, "Stochastic Approach for System Identification using Machine Learning", Dynamics of Systems, Mechanisms and Machines (Dynamics), IEEE, pp. 1-4, 2019.

[20] E. Solak, R. Murray-Smith, W.E. Leithead, D.J. Leith, C.E. Rasmussen, "Derivative Observations in Gaussian Process Models of Dynamic Systems", Advances in Neural Information Processing Systems, MIT Press, pp. 529-536, 2003.

[21] C.E. Rasmussen, C.K. Williams, "Gaussian Processes for Machine Learning", MIT Press, Vol. 1, Cambridge, MA, 2006.

[22] J. Kocijan, "Modelling and Control of Dynamic Systems Using Gaussian Process Models", Springer, 2016.

[23] D. Angeli, J. Rawlings, "Economic Optimization Using Model Predictive Control with a Terminal Cost", Annual Reviews in Control, Vol. 35, No. 2, pp. 178-186, 2011.

[24] M. Diehl, G. Bock, J. Schloder, R. Findeisen, Z. Nagy, F. Allgower, "Real-Time Optimization and Nonlinear Model Predictive Control of Processes Governed by Differential-Algebraic Equations", Journal of Process Control, Vol. 12, No. 4, pp. 577-588, 2002.

[25] B. Kouvaritakis, M. Cannon, P. Couchman, "MPC as a Tool for Sustainable Development Integrated Policy Assessment", IEEE Trans. Autom. Control, Vol. 51, No. 1, pp. 145-149, 2006.

[26] P. Patrinos, S. Trimboli, A. Bemporad, "Stochastic MPC for Real-Time Market-Based Optimal Power Dispatch", Proceedings of the 50th Conference on Decision and Control, Orlando, USA, pp. 7111-7116, 2011.

[27] G. Serale, M. Fiorentini, A. Capozzoli, D. Bernardini, A. Bemporad, "Model Predictive Control (MPC) for Enhancing Building and HVAC System Energy Efficiency: Problem Formulation, Applications and Opportunities", Energies, Vol. 11, No. 3, pp. 1-35, 2018.

[28] M. Gwerder, J. Todli, "Predictive Control for Integrated Room Automation", 8th REHVA World Congr. Building Technol., October, pp. 1-6, 2005.

[29] J.A.E. Andersson, J. Gillis, G. Horn, J.B. Rawlings, M. Diehl, "CasADi - A Software Framework for Nonlinear Optimization and Optimal Control", Mathematical Programming Computation, Vol. 11, No. 1, pp. 1-36, 2019.

[30] K.H. Ang, G. Chong, Y. Li, "PID Control System Analysis, Design, and Technology", in IEEE Transactions on Control Systems Technology, Vol. 13, No. 4, pp. 559-576, 2005.

BIOGRAPHIES



Shokhjakhon Abduattokhov was born on October 20, 1992 in Andijan, Uzbekistan. In 2011, he entered to Turin Polytechnic University in Tashkent, Uzbekistan and earned B.Sc. degree with distinction of Mechanical Engineering in 2015. From 2015 to 2016, he worked as

an engineer in GM Uzbekistan car plant. Subsequently, he was enrolled in Erasmus Mundus Joint Master Degree Program in Mathematical Modelling in Engineering and successfully defended his thesis in 2018 at University of L'aquila, Italy. He has been working as an assistant teacher of Control and Computer Engineering Department in Turin Polytechnic University in Tashkent since 2021. He has published more than 20 articles in high impact worldwide journals and conferences. His research interest is in numerical modeling, system identification, machine learning and optimal control.



Kamila Ibragimova was born in Tashkent, Uzbekistan on February 6, 1969. She received the B.Sc. and M.Sc. degrees in Computer Engineering from Tashkent State Technical University, Uzbekistan, in 1991. Currently, she is a full Professor at computer engineering

department of Tashkent University of Information Technologies, Uzbekistan. Her research interests are in the application of artificial intelligence to power system control design, dynamic load modeling. She is an author of more than 40 papers published in well-recognized journals.



Markhamat Khaydarova was born in Tashkent, Uzbekistan on March 9, 1960. She received a five-year degree in Computer Engineering from the Tashkent Pedagogic Institute, in 1987 and the Ph.D. degree in Applied Mathematics from the same university, in 1995. She is currently

a full Professor at the Tashkent University of Information Technologies, Uzbekistan. She has authored of 2 books in Electronic Trade and one book in Programming and Algorithms area. Furthermore, she has published more than 90 scientific articles and thesis. She is an Associate Editor of scientific journal of Tashkent University of Information Technologies of "TUIT Letters".



Abbos Abdurakhmanov was born in Yunusobod, Tashkent, Uzbekistan on January 18, 1963. He received the B.Sc. and M.Sc. degrees on Applied Mathematics from the Tashkent State University, Uzbekistan in 1969. From 1984 to 1990 he worked as an engineer-programmer, senior programmer, the 1st category engineer-programmer of the Institute of Cybernetics of the

Academy of Sciences of Uzbekistan. Since 2018, he has been a senior lecturer of the Department of Fundamentals of Informatics at the Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, Uzbekistan. During his working period, he has written more than 20 papers and one book in computer engineering field. His research interest is numerical optimization, convex optimization, modeling dynamical systems and optimal control.