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ENGINEERING APPLICATIONS FOR ELECTROMAGNETICALLY-MECHANICAL FIELDS

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Abstract- This paper is presenting some of the engineering applications related to transformation of electromagnetic energy into mechanical energy. The numerical determination of the interaction forces between current-carrying conductors and the carrying force of an electromagnet are interesting for the reader because different numerical calculation methods are used, and the results are compared with the measurements performed. The conclusions are important both from theoretically and practically point of view.

Keywords: Electromagnetic Field Laws, Electromagnetic Energy, Finite Element Method, Interaction Forces, FEM, Engineering Applications, Electromagnetic Field Synthesis, Computation and Analysis.

1. INTRODUCTION

The study of electromagnetic phenomena is done indirectly, through mechanical, chemical, optical or thermal side effects [1]. Among these reproducible effects, the mechanical forces and moments or the ponderomotive interactions exerted on the bodies in a closed domain together with the electromagnetic field prove the transformation of the electromagnetic energy into mechanical energy respecting the law of energy conservation [2]. If the total energy of the electromagnetic field is known, its interaction with any physical system consisting of bodies can be described by means of electromagnetic forces [3].

The second part of the paper makes, at the beginning, a synthetic presentation of the quantities and of the general laws of the macroscopic theory of the electromagnetic field that will present the readers with these theoretical notions. Then the electromagnetic energy theorem is stated and proved, which states that the transfer of electromagnetic field power to a stationary domain through its boundary is the sum of the power transferred to the bodies in the field and the rate of increase of electromagnetic field energy in the considered field. So, the decrease of the electromagnetic energy in the elementary unit of time in the considered field is equal to the sum between the power developed in the electrical conduction process and the power transformed into mechanical energy. The additive separation of the electromagnetic force, in volume density of the electric and magnetic force, will be the basis of the applications developed in this paper [4-16].

In many engineering applications it is necessary to determine the interaction forces between two parallel conductors, rectilinear, located at a distance from each other and circulated by currents. Also, the calculation of the carrying force of an electromagnet that is used in numerous electromagnetic and electromechanical equipment is important from a theoretical and practical point of view. Therefore, the third part of this paper will briefly describe the constructive and functional characteristics of some electromagnetic equipment, such as the transformer, the electromagnet and the aerial lines of the electric power system [17].

Advanced numerical calculation methods of the electromagnetic field and of the electromagnetic energy will be described in the fourth part of the paper [18-25]. In engineering practice, an electromagnetic field analysis problem consists in determining the electromagnetic field corresponding to given conditions of uniqueness. The finite element method is based on the approximation of partial differential equations as well as approximation of the solutions by expressions defined on a partition of the field to be studied in disjoint elements, called "finite elements", which give the name of the method. In the traditional approach to the finite difference method, the equations of the electromagnetic field in their differential form are used. From the point of view of the concrete approach, the finite element method has two main variants: the Galerkin method, based on the use of a so-called "weak form" of the field equations, and the convenient discretization of the domain and the Ritz method, in which the solution is obtained by minimizing a certain functional [26]. The FEMM programming environment has all the modules necessary to use the finite element method, for any linear and nonlinear structures.

The FEMM program allows the numerical simulation by the finite element method of electromagnetic field problems for low frequency (industrial) electromagnetic devices and high frequency electromagnetic devices. The first class includes devices, appliances, machines, etc. working in low frequency alternating electromagnetic field such as electric transformers, asynchronous motors, electromagnets etc. For the first class there is a wide variety of program packages based on the finite element method in 2D (plane simulation) and 3D (three-dimensional simulation) space. In this part of the paper, applications for determining the interaction forces between the conductors of the electricity transmission lines and between the conductors of some power transformers will be developed in the FEMM programming environment. The calculation of the load force for an electromagnet used in electromagnetic equipment will be another important application [27-34]. The values obtained from the calculation in the FEMM programming environment will be compared with the results of the measurements that the authors performed on the electromagnetic equipment used [35-42]. The calculated errors will be analyzed. The paper ends with conclusions on the applications developed and an up-to-date bibliography.

2. MACROSCOPIC ELECTROMAGNETIC FIELD THEORY - PHYSICAL QUANTITIES AND GENERAL LAWS

2.1. Macroscopic Electromagnetic Field Theory Physical Quantities and General Laws

2.1.1. Electric Flux Law (Gauss's Law on The Electric Flux)

Statement: Electric flux Ψ_Σ through any closed surface Σ is equal at any moment to the electric charge q_{V_Σ} in domain V_Σ bounded by the surface Σ (Figure 1).

$$\Psi_\Sigma = q_{V_\Sigma} \qquad (1)$$

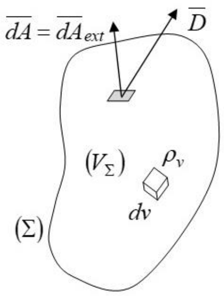

Figure 1. General domain for the electric flux law

If electric charge from V_Σ has the volume distribution density a ρ_V :

$$\rho_V = dq_{V_\Sigma} / dv \qquad (2)$$

then, applying Gauss-Ostrogradski relation, one obtains the integral form electric flux law:

$$\int_\Sigma \overline{D} d\overline{A} = \int_{V_\Sigma} \operatorname{div} \overline{D} dv = \int_{V_\Sigma} \rho_V dv \qquad (3)$$

where, being true for any arbitrary considered domain V_Σ imposes the equality of the integrands.

$$\operatorname{div} \overline{D} = \rho_V \qquad (4)$$

The relation (4) represents the local form the electric flux law for continuity domains. Electric flux law emphasizes one of the causes that generate electric field, namely charged bodies (Figure 2).

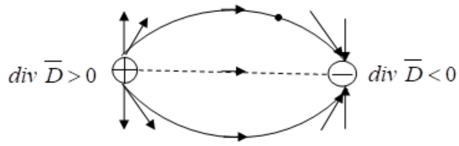

Figure 2. Charged bodies

2.1.2. Magnetic Flux Law

Statement: Magnetic flux Φ_Σ through any closed surface Σ is zero in any moment:

$$\Phi_\Sigma = 0 \qquad (5)$$

Replacing in Equation (5) the expression of the magnetic flux, one obtains the integral form of the magnetic flux law:

$$\int_\Sigma \overline{B} d\overline{A} = \int_{V_\Sigma} \operatorname{div} \overline{B} dv = 0 \qquad (6)$$

Because this relation is valid for any space domain (V_Σ), it results the local form for continuity domains of the magnetic flux law:

$$\operatorname{div} \overline{B} = 0 \qquad (7)$$

According to relation (7), the magnetic flux density field vector is a solenoidal one (without sources). This fact underlines on one hand the non-existence of the magnetic charges similar to electric charges and, on the other hand, inexistence of some points - extremity of magnetic field lines. Therefore, magnetic field lines are not open curves.

An immediate consequence of the magnetic flux law magnetic flux through any open surface bounded by the same closed curve is the same. In order to prove this statement one will consider an arbitrary closed curve Γ, and two arbitrary open surfaces $S_{\Gamma,j}$ and $S_{\Gamma,k}$, which rest on curve Γ, as Figure 3.

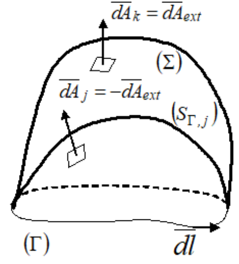

Figure 3. General domain for the magnetic flux law

By convention the line-oriented element is associated with each of the area-oriented elements, and according to the right drill rule, the relation (5) for the closed curve, reunion of the open surfaces is written. In these conditions, the integral will be:

$$\int_{\Sigma} \bar{B} d\bar{A}_{ext} = \int_{S_{\Gamma,j}} \bar{B} d\bar{A}_{ext} + \int_{S_{\Gamma,k}} \bar{B} d\bar{A}_{ext} = \int_{S_{\Gamma,j}} \bar{B} d\bar{A}_j - \int_{S_{\Gamma,k}} \bar{B} d\bar{A}_k = 0 \quad (8)$$

therefore,

$$\int_{S_{\Gamma,j}} \bar{B} d\bar{A}_k = \int_{S_{\Gamma,k}} \bar{B} d\bar{A}_k = \int_{S_{\Gamma}} \bar{B} d\bar{A}_k \quad (9)$$

For any open surface which sits on the closed curve, which shows that the magnetic flux has a unique value through all open surfaces bounded by the same closed curve. Moreover, the vector identity:

$$\operatorname{div}\left(\operatorname{rot}\bar{A}\right) = 0 \quad (10)$$

Allows the introduction of a new quantity, called magnetic vector potential, and denoted by $\bar{A}$ with relation:

$$\operatorname{rot}\bar{A} = \bar{B} \quad (11)$$

As vector $\bar{A}$ is uniquely determined only if we know its divergence, it is common that in stationary regime to adopt the calibration condition

$$\operatorname{div}\bar{A} = 0 \quad (12)$$

such that the field vector $\bar{A}$ to be a solenoidal one too.

Applying Stokes' theorem, the magnetic flux which flows through an arbitrary open surface S_Γ, which sits on a closed curve Γ, can be expressed by line integral of the of the magnetic potential vector on the curve Γ:

$$\Phi_{S_\Gamma} = \int_{S_\Gamma} \bar{B} d\bar{A} = \int_{S_\Gamma} \operatorname{rot}\bar{A} d\bar{A} = \int_{\Gamma} \bar{A} d\bar{l} \quad (13)$$

This shows that magnetic flux through an open surface depends only on the closed curve that bounds it.

2.1.3. Electromagnetic Induction Law (Faraday's Law)

Statement: The electromotive force (emf) u_Γ along a closed curve Γ is equal to the rate of decrease (in time) of the magnetic flux Φ_{S_Γ} across any surface S_Γ bordered by the closed curve Γ:

$$u_\Gamma = -\frac{d\Phi_{S_\Gamma}}{dt} \quad (14)$$

Considering the definition of the emf and of the magnetic flux it results the integral form of the law

$$\int_\Gamma \bar{E} d\bar{l} = -\frac{d}{dt} \int_{S_\Gamma} B d\bar{A} \quad (15)$$

The electromagnetic induction law has the above form only with the condition that the reference direction of the closed curve Γ (the reference direction of the oriented line element $d\bar{l}$) and the direction of the normal to the surface S_Γ (with the oriented area element $d\bar{A}$) are associated according to right corkscrew rule (Figure 4).

For moving media, the integration domains follow the bodies in their movement, and the derivative with respect to time of the magnetic flux is a substantial derivative and it is computed using the following relation:

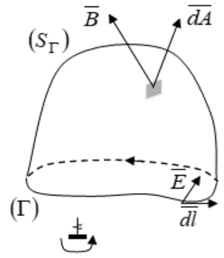

Figure 4. General domain for the electromagnetic flux law

$$\frac{d}{dt} \int_{S_\Gamma} \bar{B} d\bar{A} = \int_{S_\Gamma} \left[\frac{\partial \bar{B}}{\partial t} + \bar{w} \operatorname{div}\bar{B} + \operatorname{rot}\left(\bar{B} \times \bar{w}\right) \right] d\bar{A} \quad (16)$$

where, $\bar{w}$ is the local speed vector of the medium.

Using Stokes relation, it results that

$$\int_\Gamma \bar{E} d\bar{l} = \int_{S_\Gamma} \left(\operatorname{rot}\bar{E}\right) d\bar{A} \quad (17)$$

and,

$$\int_{S_\Gamma} \left[\operatorname{rot}\left(\bar{B} \times \bar{w}\right) \right] d\bar{A} = \int_\Gamma \left(\bar{B} \times \bar{w}\right) d\bar{l} \quad (18)$$

Considering the local form of the magnetic flux law, one obtains a new integral form of the electromagnetic induction law:

$$u_\Gamma = \int_\Gamma \bar{E} d\bar{l} = -\int_{S_\Gamma} \frac{\partial \bar{B}}{\partial t} d\bar{A} - \int_\Gamma \left(\bar{B} \times \bar{w}\right) d\bar{l} \quad (19)$$

Relation (19) emphasizes the physical significance of the law: the time variable magnetic field produces (induces) an electric field by the electromagnetic induction phenomena. Therefore, the electromagnetic induction is a physical phenomenon, unlike electric displacement $\bar{D}$ and magnetic induction $\bar{B}$ which are physical quantities.

Moreover, relation (19) allows the decomposition of the emf into two components:

$$u_\Gamma = u_t + u_m \quad (20)$$

with

$$u_t = -\int_{S_\Gamma} \frac{\partial \bar{B}}{\partial t} d\bar{A} = \int_{S_\Gamma} -\frac{\partial \bar{B}}{\partial t} d\bar{A} \quad (21)$$

called emf induced by transformation and, respectively.

$$u_m = -\int_\Gamma \left(\bar{B} \times \bar{w}\right) d\bar{l} = \int_\Gamma \left(\bar{w} \times \bar{B}\right) d\bar{l} \quad (22)$$

called emf induced by movement.

In stationary regime, the electromagnetic induction law becomes the theorem of the stationary electric potential, which has the local form:

$$\operatorname{rot}\bar{E} = 0 \quad (23)$$

and, respectively, the integral form:

$$u_\Gamma = \int_\Gamma \bar{E} d\bar{l} = 0 \quad (24)$$

The local form (23) of electromagnetic induction law in stationary regime shows that, in this regime, the vector field $\bar{E}$ is non-rotational and, according to the vector identity which states that the curl of the gradient for any scalar field is zero, it can be introduced the scalar quantity V, called electric potential, with relation:

$$\bar{E} = -\operatorname{grad}V \quad (25)$$

The integral form of the electromagnetic induction law in stationary regime allows the demonstration of the following theorem: the voltage drop between two points M_k and M_j, from space (arbitrary points) does not depend on the paths (integration curve) between them.

Indeed (Figure 5), let consider two paths (arbitrary) between M_k and M_j, represented by the open curves C_1 and C_2, whose reunion is the closed curve Γ. According to relation (24) it results:

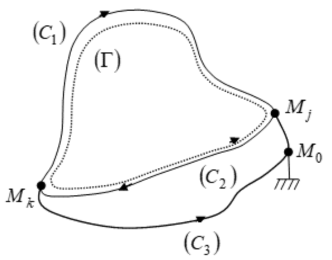

Figure 5. Voltage drops between two points

$$u_\Gamma = \oint_{(\Gamma)} \bar{E}d\bar{l} = \int_{\substack{M_k \\ (C_1)}}^{M_j} \bar{E}d\bar{l} + \int_{\substack{M_j \\ (C_2)}}^{M_k} \bar{E}d\bar{l} = \int_{\substack{M_k \\ (C_1)}}^{M_j} \bar{E}d\bar{l} - \int_{\substack{M_k \\ (C_2)}}^{M_j} \bar{E}d\bar{l} = 0 \quad (26)$$

that is

$$\int_{\substack{M_k \\ (C_1)}}^{M_j} \bar{E}d\bar{l} = \int_{\substack{M_k \\ (C_2)}}^{M_j} \bar{E}d\bar{l} = u_{M_k,M_j} \quad (27)$$

The electric tension is therefore a scalar physical quantity referring to an ordered pair consisting of two points from the space, while the electric potential is a physical scalar quantity associated to each point in the space.

As relation (25) is a differential type relation, it means that electric potential is defined up to an arbitrary additive constant. This constant represents the value $V(M_0) = V_{M_0}$, arbitrary, of potential M_0 from the space, arbitrary, considered to be reference value for all potentials. Then, the potential of any point from the space, for example M_j, is:

$$V(M_j) = V_{M_j} = V_{M_0} - \int_{M_0}^{M_j} \bar{E}d\bar{l} \quad (28)$$

If one considers $V_{M_0} = 0$, it is said that point M_0 is "grounded" or "earthed" and in an electric circuit it has a specific symbol (Figure 5).

In these conditions

$$V(M_j) = V_{M_j} = \int_{Mj}^{M_0} \bar{E}d\bar{l} \quad (29)$$

2.1.4. The Magnetic Circuit Law (Ampere)

Statement: The magnetomotive force (mmf) u_{m_Γ} along any closed curve Γ is equal to the sum between the conduction electric current i_{S_Γ} through an open surface S_Γ, arbitrary, bordered by the closed curve Γ and the time derivative of the electric flux over the same surface S_Γ:

$$u_{m_r} = i_{S_\Gamma} + \frac{d\Psi_{S_\Gamma}}{dt} \quad (30)$$

The term i_{S_Γ} is also denoted by Θ_{S_Γ}. By rewriting relation (30) one obtains the integral form of the law:

$$\int_\Gamma \bar{H} \times d\bar{l} = \int_{S_\Gamma} \bar{J} \times d\bar{A} + \frac{d}{dt}\int_{S_\Gamma} \bar{D} \times d\bar{A} \quad (31)$$

Which has the above presented form only if the association between the reference direction of the orientation the oriented line element $d\bar{l}$ and the oriented area element $d\bar{A}$ is according to the corkscrew rule (as for the electromagnetic induction law).

For moving media, the integration domains follow the bodies in their movement, following a similar path as the one described for the electromagnetic induction law, and the integral developed form of magnetic circuit law is obtained:

$$\begin{aligned} \int_\Gamma \bar{H}d\bar{l} &= \int_{S_\Gamma} \left(\text{rot}\bar{H}\right)d\bar{A} = \int_{S_\Gamma} \bar{J}d\bar{A} + \\ &+ \int_{S_\Gamma} \left[\frac{\partial \bar{D}}{\partial t} + \bar{w}\text{div}\bar{D} + \text{rot}\left(\bar{D} \times \bar{w}\right)\right]d\bar{A} = \int_{S_\Gamma} \bar{J}d\bar{A} + \\ &+ \int_{S_\Gamma} \frac{\partial \bar{D}}{\partial t}d\bar{A} + \int_{S_\Gamma} \left(\bar{w}\text{div}\bar{D}\right)d\bar{A} + \int_{S_\Gamma} \left[\text{rot}\left(\bar{D} \times \bar{w}\right)\right]d\bar{A} \end{aligned} \quad (32)$$

$$u_{m_\Gamma} = i_{S_\Gamma} + i_{d_{S_\Gamma}} + i_{v_{S_\Gamma}} + i_{R_{S_\Gamma}} \quad (33)$$

On the right side of Equation (33) we have four terms:

- The conduction current:

$$i_{S_\Gamma} = \int_{S_\Gamma} \bar{J}d\bar{A} \quad (34)$$

- The displacement current:

$$i_{d_{S_\Gamma}} = \int_{S_\Gamma} \frac{\partial \bar{D}}{\partial t}d\bar{A} \quad (35)$$

where,

$$\frac{\partial \bar{D}}{\partial t} = \bar{J}_d \quad (36)$$

is the displacement current density;

- The convection current:

$$i_{v_{S_\Gamma}} = \int_{S_\Gamma} (\bar{w} \times \text{div}\bar{D})d\bar{A} = \int_{S_\Gamma} \bar{w} \times \rho_v \times d\bar{A} \quad (37)$$

where,

$$\bar{w} \times \rho_v = \bar{J}_v \quad (38)$$

is the convection current density;

- Roentgen current

$$i_{R_{S_\Gamma}} = \int_{(S_\Gamma)} \left[\text{rot}(\bar{D} \times \bar{w})\right]d\bar{A} = \int_{(\Gamma)} (\bar{D} \times \bar{w})\,d\bar{l} \quad (39)$$

The magnetic circuit law emphasizes two causes that can generate the magnetic field: conductive bodies transited by conduction currents and/or time variable electric fields (by displacement currents, convection and Roentgen currents).

For continuity domains, the equality (32), true for any surface S_Γ , leads to:

$$\text{rot}\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} + \bar{w}\rho_v + \text{rot}(\bar{D} \times \bar{w}) \quad (40)$$

relation that represents the local form of the magnetic circuit law.

For stationary media $\bar{w} = 0$ the local form becomes:

$$\text{rot}\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} \quad (41)$$

and it shows that the closed lines of the magnetic field surround the conductors transited by conduction currents, respectively the lines of the time variable electric field that generate them.

In steady state regime the magnetic circuit law becomes the Ampère theorem, and it has the local form:

$$\text{rot}\,\bar{H} = \bar{J} \quad (42)$$

and, respectively, the global form

$$u_{m_\Gamma} = i_{s_\Gamma} \quad (43)$$

In the particular cases of the point from the space for which the vector field $\bar{H}$ is a non-rotational ($\text{rot}\bar{H} = 0$), according to the vector identity which states that the curl of the gradient of any scalar field is zero, the scalar quantity V_m , can be introduced, called scalar magnetic potential, using the relation [3, 13, 14]:

$$\bar{H} = -\text{grad}V_m \quad (44)$$

2.2. Electromagnetic Energy Theorem

According to the nature of the state quantities on which each of the additive terms in the general expression of the total energy of a system depends, the energy is mechanical, electrical, magnetic, etc., each of these terms being a form of energy. Mechanical work is not a form of energy, because it does not characterize physical systems, but their interactions and their transformations; therefore, the heat that a body exchanges with its exterior is not a form of energy either.

The close-up action theory states that physical actions are localized, and because energy is a function of the state of physical systems, it follows that energy depends exclusively on local state quantities. In this sense, the most general expression of energy is an algebraic sum of the elementary energies (energy densities) contained in the field. The part of the total energy of a physical system that depends only on the relative positions in relation to the outside is called potential energy; according to the quantities of state that intervene in its expression, the potential energy is electrical, mechanical, etc. The part of the total energy of a physical system that depends only on its internal state quantities is called internal or internal energy. If the physical system is isolated, its total energy is equal to its inner energy. In the macroscopic theory of the electromagnetic field, it is considered that the inner energy is susceptible exclusively to continuous variation.

Let us assume a system of immobile, linear and isotropic bodies, without permanent electric polarization and permanent magnetization, which is in a domain v_Σ bounded by the closed surface Σ in interaction with an electromagnetic field. Therefore, the bodies are characterized by material quantities ε and μ independent of the field.

We define the electromagnetic energy as the term in the most general expression of the total energy of a physical system that depends exclusively on the quantities of electrical and magnetic states of the system. It is obvious that this form of energy is always associated with the electromagnetic field and is canceled once the electromagnetic field disappears. Therefore, for the linear fields, one chooses - if possible - as a reference state in relation to which the electromagnetic energy is calculated, the state characterized by zero values of the local state quantities $\bar{E}$, $\bar{D}$, $\bar{H}$ and $\bar{B}$ of the electric and magnetic field.

To the physical system consisting of bodies and the electromagnetic field from the domain v_Σ , the energy conservation law for an elementary transformation is: in a time, interval *dt*, the decrease ($-dW$) of the electromagnetic field energy in the considered domain is equal to the sum of the electromagnetic energy dW_Σ leaving through the surface Σ from the domain v_Σ and the energy dW_t transformed into other forms of energy in the domain v_Σ:

$$-dW = dW_\Sigma + dW_t \quad (45)$$

If (45) is divided at elementary time intervals dt, the following equivalent relationship in powers is obtained:

$$\frac{-dW}{dt} = P_\Sigma + P_t \quad (46)$$

where, $P_\Sigma = \frac{dW_\Sigma}{dt}$ is the electromagnetic power which is transmitted inside the considered domain through the surface Σ and, respectively $P_t = \frac{dW_t}{dt}$ is the power transformed into other forms of energy, non-electromagnetic, inside the considered domain.

The transformation of electromagnetic energy into other forms of energy can be done through the process of electrical conduction, through the variation of electrical polarization, through the variation of the magnetization of bodies with hysteresis and through the movement of bodies.

Because, in the considered field, the bodies have linear and constant properties and ε, μ and σ are immobile, it results that the only possibility of transformation of the electromagnetic energy remains in the process of electrical conduction. According to the law of transformation of electromagnetic energy in the conduction process, it results that:

$$P_t = P_J = \int_{v_\Sigma} P_J\,dv = \int_{v_\Sigma} \bar{E}\bar{J}dv \quad (47)$$

Similarly, the electromagnetic power transmitted outside the considered domain through by the surface Σ can be defined as the flux of a power density vector through this closed surface:

$$P_\Sigma = \int_\Sigma \bar{S}d\bar{A} \quad (48)$$

the vector *S* being called the Poynting vector.

The Poynting vector is defined with the approximation of an arbitrary solenoidal field. Therefore, the vector:

$\bar{S}' = \bar{S} + \text{rot}\bar{G}$

where, $\bar{G}$ is an arbitrary vector, has the same flux through a closed surface. This does not introduce an un-determination of the considered surface Σ because the Poynting vector intervenes by definition only in the integrals on closed surfaces, i.e., through its divergence.

In concordance with the concept regarding the close-up theory of the actions, one introduces the concept of volume density of the electromagnetic energy *w*, to consider its space distribution in the volume Δv:

$$w = \lim_{\Delta v \to 0} \frac{\Delta W}{dv} = \frac{dW}{dv} \tag{49}$$

Using (47), (48) and (49), the Equation (46) becomes:

$$-\frac{d}{dt}\int_{v_\Sigma} w dv = \int_{v_\Sigma} \bar{E}\bar{J} dv + \int_{\Sigma} \bar{S} d\bar{A} \tag{50}$$

The first term of the right side of the above relation is calculated, i.e., the volume density of the power transformed in the conduction process, taking into account the law of the magnetic circuit and the law of electromagnetic induction, as follows:

$$\begin{gathered} p_J = \bar{E}\bar{J} = \bar{E}\left(\text{rot}\bar{H} - \frac{\partial \bar{D}}{\partial t}\right) = \text{div}\left(\bar{H} \times \bar{E}\right) + \\ +\bar{H}\text{rot}\bar{E} - \bar{E}\frac{\partial \bar{D}}{\partial t} = \text{div}\left(\bar{H} \times \bar{E}\right) + \left(\bar{H}\frac{\partial \bar{B}}{\partial t} + \bar{E}\frac{\partial \bar{D}}{\partial t}\right) \end{gathered} \tag{51}$$

The medium is linear, therefore:

$$\bar{E}\frac{\partial \bar{D}}{\partial t} = \varepsilon \bar{E}\frac{\partial \bar{E}}{\partial t} = \varepsilon \frac{\partial\left(\bar{E}^2\right)}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\bar{E}\bar{D}}{2}\right)$$

and

$$\bar{H}\frac{\partial \bar{B}}{\partial t} = \mu \bar{H}\frac{\partial \bar{H}}{\partial t} = \mu \frac{\partial(\bar{H}^2)}{\partial t} = \frac{\partial}{\partial t}(\frac{\bar{H}\bar{B}}{2})$$

Finally, the total power transformed in the conduction process in the whole domain v_Σ is equal to:

$$P_J = \int_{v_\Sigma} p_j dv = \int_{v_\Sigma} \text{div}(\bar{H} \times \bar{E}) dv - \int_{v_\Sigma} \frac{\partial}{\partial t}(\frac{\bar{E}\bar{D}}{2} + \frac{\bar{H}\bar{B}}{2}) dv$$

If the Gauss-Ostrogradski theorem is applied to the first term in the right member of the previous relation and the terms are arranged to compare them with those in the relation (51), we obtain:

$$-\int_{v_\Sigma} \frac{\partial}{\partial t}(\frac{\bar{E}\bar{D}}{2} + \frac{\bar{H}\bar{B}}{2}) dv = \int_{v_\Sigma} p_j dv + \int_{\Sigma} (\bar{E} \times \bar{H}) d\bar{A} \tag{52}$$

Because the domain v_Σ is arbitrary chosen, by comparing the relationships (51) and (52), it results the expression of the volume density of the electromagnetic energy:

$$w = \frac{\bar{E}\bar{D}}{2} + \frac{\bar{H}\bar{B}}{2} \tag{53}$$

respectively of the flux density of the power transmitted through electromagnetic field (Poynting vector):

$$\bar{S} = \bar{E} \times \bar{H} \tag{54}$$

i.e., the electromagnetic field theorem. Intuitively the two terms form (53) of the energy volume density are called volume density of electric energy:

$$W_e = \frac{\bar{E}\bar{D}}{2} \tag{55}$$

and the volume density of the magnetic energy:

$$W_m = \frac{\bar{H}\bar{B}}{2} \tag{56}$$

The two names being valid only in the static regime of the electromagnetic field, the only one in which the two fields are studied separately, without the electric and magnetic quantities and phenomena influencing each other.

2.3. Theorem of Ponderomotive Actions Developed by Electromagnetic Field

It is considered an infinitely large domain, v_∞ "filled" with a linear, isotropic environment, without permanent electrical polarization and permanent magnetization, in which there is a system of bodies that have a gradual variation of mass density at the surface and not a sudden variation of through clear borders. Therefore, mass density is a continuous function of point and time, $\tau = \tau(\bar{r}, t)$.

The environment is supposed to be inhomogeneous in terms of electrical and magnetic properties and the electrical and magnetic permittivity depend on point and time, exclusively as a function of mass density:

$\varepsilon = \varepsilon\left(\bar{r}, t\right) = \varepsilon\left(\tau\right)$

$\mu = \mu\left(\bar{r}, t\right) = \mu\left(\tau\right)$

The environment is moving, and the bodies are deformable, their movement being described by the continuous field of velocities: $\bar{v} = \bar{v}(\bar{r}, t)$.

The determination of the force density *f* exerted by the electromagnetic field on the body system, starts from the application of the law of energy conservation, on the closed system formed by bodies and the electromagnetic field, which is considered to be extended to infinity. In this case, the rapid decrease of the electric field intensity and the decrease of the magnetic field intensity with distance, lead to the conclusion that the Poynting vector cancels indefinitely and there is no power transmission outside the considered range, through its boundary. Therefore, the balance of power becomes:

$$-\frac{dW_{v_\infty}}{dt} = P_{J,v_\infty} + P_{M,v_\infty}$$

and shows that the decrease of the electromagnetic energy in the elementary unit of time in the considered field is equal to the sum between the power developed in the electrical conduction process (P_{J,v_∞}) and the power transformed into mechanical energy (P_{M,v_∞}).

The expression of the electromagnetic energy theorem is used:

$$W_{v_\infty} = \int_{v_\infty} (\frac{\bar{E}\bar{D}}{2} + \frac{\bar{H}\bar{B}}{2}) dv_\infty$$

relation of the law of energy transformation in the process of electrical conduction:

$$P_{J,v_\infty} = \int_{v_\infty} \bar{E}\bar{J}dv_\infty$$

and the expression of the mechanical power as a function of the force density f and the velocity v:

$$P_{M,v_\infty} = \int_{v_\infty} \bar{f}\,\bar{v}dv_\infty$$

finally obtaining an equation of the form:

$$-\int_{v_\infty} \left(\frac{\bar{E}\bar{D}}{2} + \frac{\bar{H}\bar{B}}{2}\right)dv_\infty = \int_{v_\infty} \bar{E}\bar{J}dv_\infty + \int_{v_\infty} \bar{f}\,\bar{v}dv_\infty$$

The development of this relationship and the calculation of the volume density of the force f are quite laborious. Therefore, only the conclusion of this demonstration is presented, which additively separates the force density f into two types of force densities. So, one obtains:

$$\bar{f} = \bar{f}_e + \bar{f}_m$$

The volume density of the electric force is:

$$\bar{f}_e = \rho_v\bar{E} - \frac{1}{2}E^2\text{grad}\varepsilon + \frac{1}{2}\text{grad}(E^2\frac{d_\varepsilon}{d_\tau}\tau) - \bar{D}\times\text{rot}\bar{E} \quad (57)$$

The volume density of the magnetic force is:

$$\bar{f}_m = \bar{J}\times\bar{B} - \frac{1}{2}H^2\text{grad}\mu + \frac{1}{2}\text{grad}(H^2\frac{d_\mu}{d_\tau}\tau) + \frac{d_f\bar{D}}{dt}\times\bar{B} \quad (58)$$

where, the terms in each right member of the previous relations are interpreted as representing different forms of action of the electromagnetic field on the bodies, as follows:

- The $\rho_v\bar{E}$ is the density of the force exerted by the electromagnetic field on the electrically charged bodies;
- $\bar{J}\times\bar{B}$ is the density of the force exerted by the electromagnetic field on the bodies in the electro-kinetic state;
- $-\frac{1}{2}E^2\text{grad}\varepsilon$ and $-\frac{1}{2}H^2\text{grad}\mu$ is the density of the force exerted by the electromagnetic field on the inhomogeneous bodies from the point of view of permittivity or permeability; the last two terms in relations (57) and (58) are negligible for sufficiently slow variations of the electromagnetic field.

These densities of forces, also called equivalent densities of forces, because they determine only the total force exerted on a body by the electromagnetic field and not its exact location, lead to the calculation of the total force F exerted on a body, which is the only experimentally observable quantity [3], [13-15]:

$$\bar{F} = \int_{v_{\text{corp}}} \bar{f}dv = \int_{v_{\text{corp}}} \left(\bar{f}_e + \bar{f}_m\right)dv \quad (59)$$

3. ELECTROMAGNETIC EQUIPMENT

3.1. Energies and Forces in Electromagnetic Equipment

The last two general theorems presented in the previous Section, the theorem of the electromagnetic energy and the theorem of the ponderomotive actions developed by the electromagnetic field, lead to the determination of the expressions of energies and forces in the stationary electrostatic and magnetic field.

3.1.1. Electrostatic And Magnetic Quasi-Stationary Field Energies

Starting from the integration of the relation (55) on a finite domain from space v_Σ, considered as a linear, homogeneous and isotropic dielectric medium, uncharged and electrically non-polarized in which there are n immobile conductors, charged with electric charges Q_k with potentials V_k. Applying the local forms of electric flux law, it is shown that the total electrostatic energy of the electrostatic field created by the n conductor system is equal to the half-sum of the products between the potentials and charges of the conductors:

$$W_e = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k \quad (60)$$

For example, the electrostatic energy of a capacitor of capacitance C, assimilated to a two-conductor system with equal charges of opposite signs $Q_1 = Q$, $Q_2 = -Q$ and with potentials V_1, V_2 is:

$$W_e = \frac{1}{2}(V_1Q - V_2Q) = \frac{1}{2}QU = \frac{1}{2}CU^2 = \frac{1}{2}\frac{Q^2}{C} \quad (61)$$

Similarly, from relation (56), it is shown that the total energy of the quasi-stationary magnetic field of a system of n filiform, immobile conductors, crossed by electric conducting currents i_k and located in a finite domain and a linear magnetic medium, homogeneous and isotropic, without permanent magnetization, is equal to the half-sum of the products between the current intensities and the magnetic fluxes Φ_k that chain the respective conductors:

$$W_m = \frac{1}{2}\sum_{k=1}^{n} i_k\Phi_k \quad (62)$$

For example, for $n = 1$, the magnetic energy of its own inductance L of a coil is obtained

$$W_m = \frac{1}{2}i\Phi = \frac{1}{2}Li^2 = \frac{1}{2}\frac{\Phi^2}{L}$$

where, for $n = 2$, i.e., for a system of two coils, of own inductances L_{11} and L_{22}, magnetically coupled by the mutual inductance $L_{12} = L_{21} = M$, the magnetic energy is obtained:

$$W_m = \frac{1}{2}L_{11}{i_1}^2 + \frac{1}{2}L_{22}{i_2}^2 + Mi_1i_2$$

3.1.2. Forces in Electrostatic Field and in Quasi-Stationary Magnetic Field

If the total energy of the electromagnetic field is known, its interaction with any physical system consisting of bodies can be described by means of electromagnetic forces.

Thus, the total force exerted by the electromagnetic field in a range v_Σ, delimited by the closed surface Σ, can be written as:

$$\bar{F} = \int_{v_\Sigma} \bar{f}dv + \bar{F}_\Sigma \quad (63)$$

where, $\overline{F}_{\Sigma}$ is the resultant of electromagnetic forces acting, in the form of mechanical stresses, on portions of the closed surface Σ, when they constitute discontinuity surfaces for field properties, and the volume density of the electromagnetic force exerted on the bodies in the considered field is defined as:

$$\overline{f} = \lim_{\Delta v \to 0} \frac{\Delta \overline{F}}{\Delta v} = \frac{d\overline{F}}{dv} \tag{64}$$

where, $\Delta \overline{F}$ the elementary force acting on the unit volume element Δv.

The volume densities of the electric and magnetic forces were from the theorem of the ponderomotive actions developed by the electromagnetic field [13-16].

3.2. Analytical Computation of Forces in Electromagnetic Equipment

3.2.1. Forces in Electrostatic Field

Starting, either from the expression of the volume density of the electric forces, or from the expression of the energy of the electrostatic field, the forces exerted by the electrostatic field on the bodies in the field can be calculated, a result which is called generalized force theorems in the electrostatic field.

We consider a system of n conductors, with charges Q_k, potentials V_k, located in an electrically uncharged dielectric medium and admitting that their relative positions can be described by m generalized coordinates $x_1, x_2, ..., x_m$ (distances, angles), the force capable of modifying only one of these coordinates, leaving the values of the others unchanged, called the generalized force X_k, which modifies the generalized coordinate x_k, can be calculated using the theorems of generalized forces in the electrostatic field, whose statements are:

- T_1: The generalized force X_k that tends to increase the generalized coordinate x_k is equal to the partial derivative with changed sign of the electrostatic energy of the system in relation to the generalized coordinate x_k, at constant charges:

$$X_k = -(\frac{\partial W_e}{\partial x_k}) Q_k = \text{ct} \tag{65}$$

- T_2: The generalized force X_k that tends to increase the generalized coordinate x_k is equal to the partial derivative of the electrostatic energy of the system in relation to the generalized coordinate x_k, at constant potentials:

$$X_k = (\frac{\partial W_e}{\partial x_k}) V_k = \text{ct} \tag{66}$$

The two theorems lead similar results, as presented in the in the following examples of analog calculation of electric forces.

➢ Example 1: Calculation of the force normally exerted on the armatures of a capacitor, of capacity C, with the voltage at terminals U, charged with the load Q and having the distance between the armatures x.

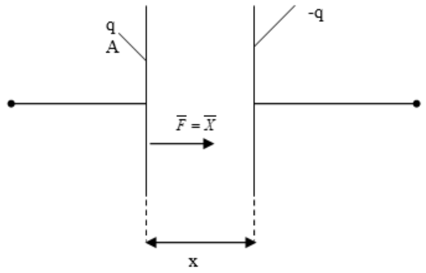

Figure 6. The field created by the capacitor armatures

The field created by the capacitor armatures is assumed to be in stationary regime, and the distance x is considered, in this case, to be the generalized coordinate (Figure 6). The area of the armatures A and the permittivity ε of the dielectric between the armatures are known.

By applying the two theorems of generalized forces in the electrostatic field, the same value for the generalized force will result, as follows:

$$T_1: \quad X = F = -\left(\frac{\partial W_e}{\partial x}\right)_{Q=ct} = -\frac{\partial}{\partial x}\left(\frac{Q^2}{2C}\right)_{Q=ct} = = \frac{U^2}{2}\frac{\partial}{\partial x}\left(\frac{\varepsilon A}{x}\right) = -\frac{U^2}{2}\frac{\varepsilon A}{x^2} \tag{67}$$

$$T_2: \quad X = F = \left(\frac{\partial W_e}{\partial x}\right)_{V=ct} = \frac{\partial}{\partial x}\left(\frac{CU^2}{2}\right)_{V=ct} = = \frac{U^2}{2}\frac{\partial C}{\partial x} = -\frac{U^2}{2}\frac{\varepsilon A}{x^2} \tag{68}$$

This example leads to the following conclusions:

1. The sign "–" in front of the expression of the normal force on the capacitor armatures shows that this force tends to decrease the distance x (being therefore an attractive force). The modulus of force is proportional to εA and inversely proportional to x^2.
2. The dependence (67) or (68) of the force of attraction between the armatures of a capacitor underlines the basic construction principle of the electrostatic measuring devices used to measure very high DC voltages (Thomson electrometer). This measuring device consists of a balance having mounted, on one arm the movable plate of a capacitor and on the other arm, a plate for weights (Figure 7).

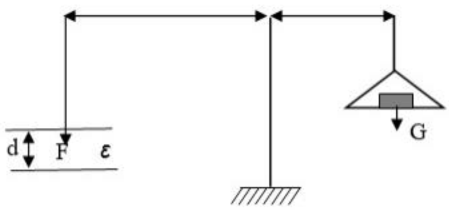

Figure 7. The measuring device

At equilibrium the moments of forces are equal:

$$Fa = Gb \quad \text{or} \quad \frac{1}{2}U^2\frac{\varepsilon A}{X^2}$$

If the equilibrium force is measured by a mechanical process, it is observed that the voltage value can be determined:

$$U = x\sqrt{\frac{2Gb}{a\varepsilon A}} \tag{69}$$

So that, by balancing this force, the distance between the reinforcements remains unchanged. In conclusion, by measuring the equilibrium force by some mechanical process, it is observed that it is dependent on U^2, being, therefore, a measure of this voltage.

3.2.2. Quasi-Stationary Magnetic Field Forces

Starting, either from the expression of the volume density of the magnetic forces, or from the expression of the quasi-stationary magnetic field energy, the forces exerted by the magnetic field on the bodies in the field can be calculated, a result called by the theorems of generalized forces in the quasi-stationary magnetic field.

Analogous to those presented in the electrostatic field, we consider n conductors crossed by currents i_k, each chained by the magnetic flux Φ_k and, assuming that there are m generalized coordinates $x_1,...,x_m$ that define the relative position of the wire conductors, the calculation of the interaction forces between these conductors is made on the basis of the theorems of the generalized forces in quasi-stationary magnetic field, knowing the energy W_m of the system:

- T_1: The generalized force X_k that tends to increase the associated generalized coordinate x_k is equal to the partial derivative of the magnetic energy of the system in relation to that generalized coordinate, taken with changed sign calculated at constant fluxes:

$$X_k = -\left(\frac{\delta W_m}{\delta x_k}\right)\Bigg|_{\Phi_k = \text{ct}} \tag{70}$$

- T_2: The generalized force X_k that tends to increase the associated generalized coordinate x_k is equal to the partial derivative of the magnetic energy of the system in relation to generalized coordinate, calculated at constant currents:

$$X_k = \left(\frac{\delta W_m}{\delta x_k}\right)\Bigg|_{i_k = \text{ct}} \tag{71}$$

As in the previous example of electrostatic forces, the two calculation theorems of magneto-stationary forces give similar results, as shown in the following example.

➢ Example 2: Computing the portent force of an electromagnet. It is considered an electromagnet, represented in Figure 8, having a fixed armature on which is located the excitation coil separated from the mobile armature by a double gap, width δ, in which there is air and which represents, in this case, the generalized coordinate.

In Figure 8, there were noted with:

l- Average fiber length of the magnetic circuit
A- Area of the magnetic circuit section
μ - Permeability of the magnetic circuit
μ_0 - Vacuum permeability
i- Current through the coil
N- Number of coils turns

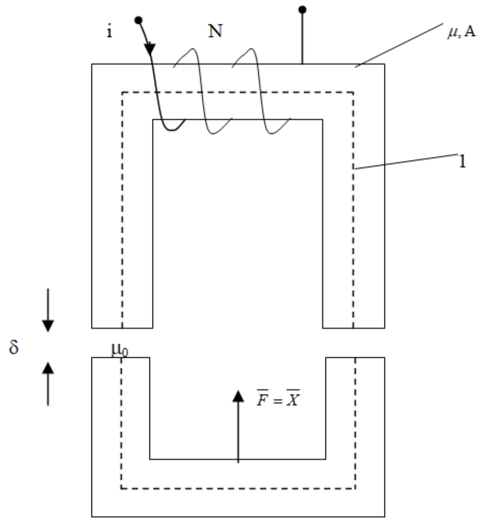

Figure 8. The electromagnet

First, calculate the reluctance R_m of the magnetic circuit:

$$R_m = \frac{2\delta}{\mu_0 A} + \frac{1}{\mu A}$$

or, in direct relation to the number of turns of the coil and the inductance of the coil,

$$L = \frac{N^2}{R_m}$$

In this case, the magnetic energy of the system can be expressed in equivalent forms:

$$W_m = \frac{1}{2}\frac{\Phi^2}{L} = \frac{1}{2}\left(\frac{\Phi}{N}\right)^2 R_m = \frac{1}{2}\Phi^2 f \times R_m = \frac{1}{2}Li^2$$

The load-bearing force exerted by the magnetic field created by the normal coil on the moving armature, is obtained with the relations of the two theorems of the generalized forces in the magnetic field, the results being similar. So, we get:

T_1:

$$X = F = -\left(\frac{\partial W_m}{\partial \delta}\right)\Phi m = ct = -\frac{1}{2}\Phi f^2 \times \frac{\partial R_m}{\partial \delta} = -\frac{B^2 A}{\mu_0} \tag{72}$$

and T_2:

$$X = F = -\left(\frac{\partial W_m}{\partial \delta}\right)_{i=ct.} = \frac{1}{2}i^2\frac{\partial L}{\partial \delta} = \frac{1}{2}i^2 N^2 \frac{\partial}{\partial \delta}\left(\frac{1}{R_m}\right) = -\frac{B^2 A}{\mu_0} \tag{73}$$

The conclusions that emerge from this calculation example are the following, [13-16]:

1. The load-bearing force is a force of attraction, which tends to reduce the air gap δ and is proportional to B^2A and inversely proportional to μ_0.
2. The principle of operation of electromagnetic measuring instruments is the calculation of this force. They are used for alternating current measurements, the force being proportional to the square of the effective value of B, which is proportional to the square of the effective value of the current that causes it.

4. SIMULATION AND RESULTS OF ENGINEERING APPLICATIONS FOR MECHANICAL AND ELECTROMAGNETIC FIELDS

In this section are analyzed three applications of numerical computation of electromagnetic forces using the results of simulations performed in FEM software environment. These computational examples were chosen from real devices that are used in complex systems to generate electromagnetic forces dependent on mechanical displacements or variable electrical quantities. The dependence between the electromagnetic and the mechanical field is synthesized, through the direct analysis of the forces generated by the electromagnetic field.

4.1. Plate Capacitor

First application refers to analysis of the force generated by the electric field of a plate capacitor and to its dependence on the distance between the plates when one is fixed and the other mobile. This device, shown in Figure 9, is used as a position sensor in complex electrical systems. The plate capacitor model was implemented in the electrostatics module of FEMM. A 2D parallel plane problem had been solved. The considered dimensions of the plates are 50 cm in length, respectively 10 cm in depth, therefore the area of the plates is 500 cm^2.

Between the plates, the considered dielectric material is air, having the relative electric permittivity (ε_r=1). One plate is considered fixed, while the other one is mobile. The distance between the plates varies between 1 cm and 10 cm. The purpose of the simulation is to compute the variation of electric force between the two plates of the capacitor depending on the distance between the plates.

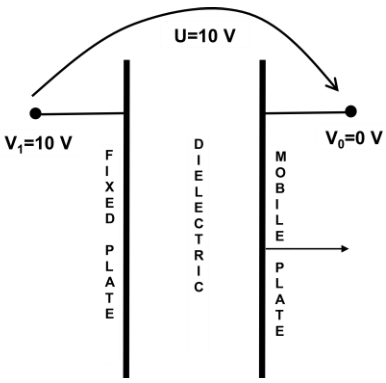

Figure 9. 2D planar model of the plate capacitor

In the pre-processing stage, one drew the geometry of the capacitor, imposed the boundary conditions, and created the mesh. To produce the electric field, one needs to impose the electric potential on the two plates or the electric charge distribution upon the plates. Because in the differential equation which describes the electrostatic regime the unknown is the electric potential, it is more convenient to impose the electric potential. Thus, on the fixed plate a potential of 10 V was imposed, while on the mobile plate a reference potential of 0 V had been considered.

Thus, a voltage of 10 V is established between the two plates. This electric potential difference creates an electric charge which accumulates on the plates.

In electrostatics all conducting bodies are equipotential, so the imposed potential is the same in each point on the plate surface. Thus, the potentials of 10 V and 0 V had been imposed as Dirichlet boundary conditions on the plates. Because FEM operates with finite computation domains, two horizontal artificial boundaries which connect the ends of plates were introduced, having Neumann boundary conditions, as seen in Figure 10. This ensures the electric field is concentrated only in the dielectric.

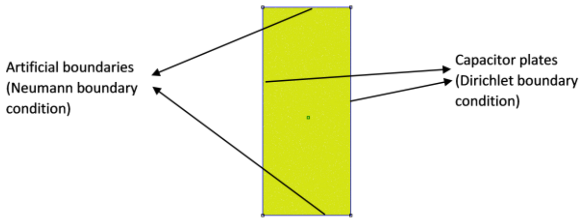

Figure 10. Mesh and boundaries

After creating the mesh that is represents in Figure 10, and solving the problem, the results obtained in the post-processing stage are shown in Figures 11, 12 and 13. Table 1 contains the values of the electric force for each distance between the plates. The force variation with the distance is shown in Figure 14 [38, 39].

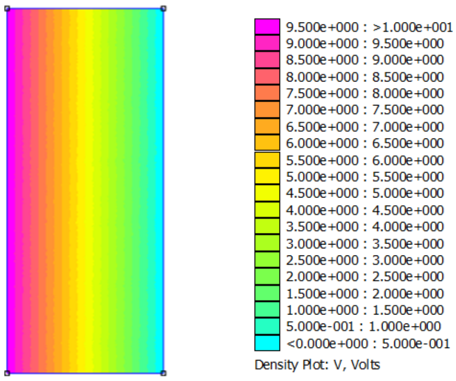

Figure 11. Density plot of electric potential

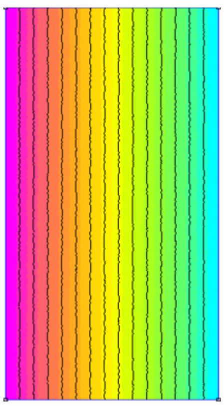

Figure 12. Electric equipotential lines

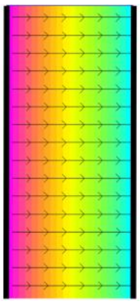

Figure 13. Electric flux density vectors

Table 1. Plate distance vs force

Plate distance [cm]	Force [nN]
10	2
9	3
8	3
7	5
6	6
5	9
4	14
3	25
2	55
1	221

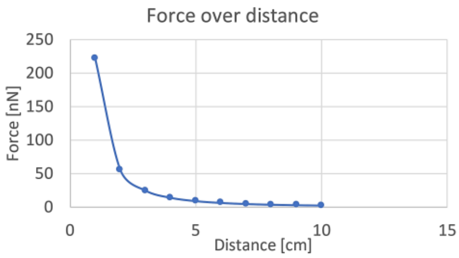

Figure 14. Force variation depending on distance

4.2. DC Electromagnet

The second application refers to analysis of the force generated by the magnetic field of an electromagnet excited by a DC current. When the air gap between the mobile and fixed iron core is variable or the value of the excitation current is changed then the force has been changed. This device is used as a position sensor or actuators in complex electrical systems.

The current excited dc electromagnet model was implemented in the magnetic module of FEMM. A 2D parallel plane problem had been solved. The considered dimensions of the electromagnet are shown in Figure 15 and are expressed in cm.

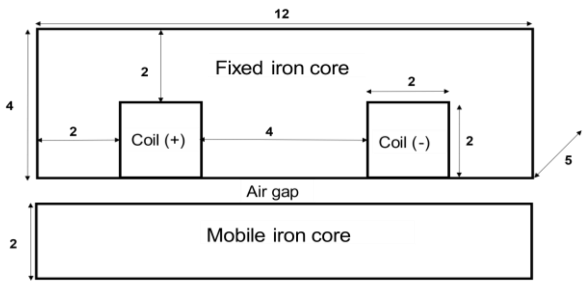

Figure 15. 2D model of electromagnet

The coil is winded on the central column and has 2000 turns. In parallel plane problems, the current density has a component on only one direction, "in the page or out of the page" (on the Oz axis). Thus, in the Coil (+) subdomain the current has a positive value (out of the page) and in the Coil (–) subdomain the current has the same value, but negative (in the page).

For the iron core, a linear ferromagnetic material had been considered, having the relative magnetic permeability μ_r=5000. The purpose of the simulation is to compute the variation of magnetic force upon the mobile iron core in two cases:

1. Fixed current, variable air gap
2. Fixed air gap, variable current

In the pre-processing stage, one drew the geometry, imposed the boundary conditions, and created the mesh, seen in Figure 16.

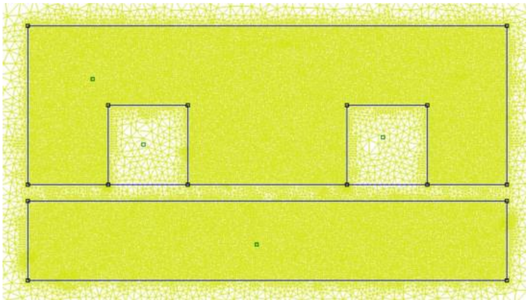

Figure 16. Mesh network

To produce the magnetic field, one imposed the current through the coil. A stationary magnetic regime (DC current) had been considered. Because FEMM operates with finite computation domains, four artificial boundaries which surround the electromagnet were introduced, having Dirichlet boundary conditions. The imposed boundary condition consists in assigning a null value to the magnetic vector potential $\bar{A}$. This ensures the magnetic field is concentrated only in the finite domain. After creating the mesh, the problem is solved and the post-processing stage is used to show the results.

4.2.1. Case 1: Fixed Current, Variable Air Gap

The current through the coil is fixed at 2 A and the air gap varies from 1 to 10 mm. For a 1 mm air gap, the results are shown in Figures 17, 18, 19, 20. Table 2 contains the values of the magnetic force for each value of the air gap. This dependence is presented in Figure 21.

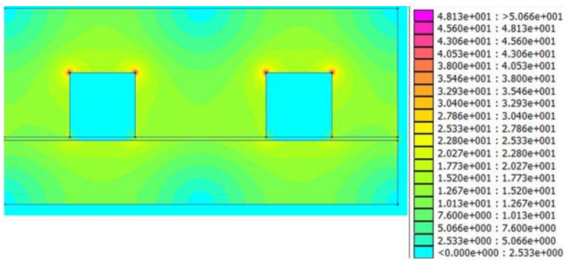

Figure 17. Magnetic flux density absolute value (legend values in T)

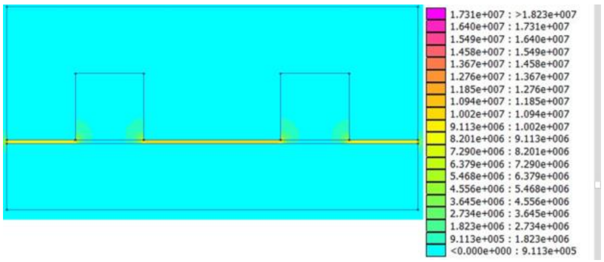

Figure 18. Magnetic field strength absolute value (legend values in A/m)

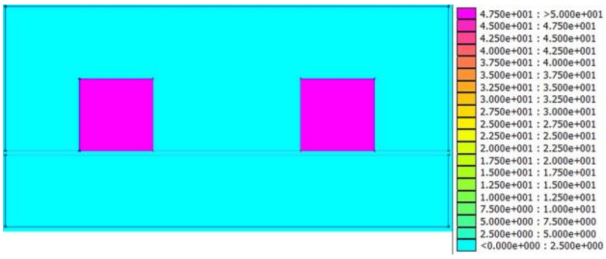

Figure 19. Current density absolute value (legend values in MA/m^2)

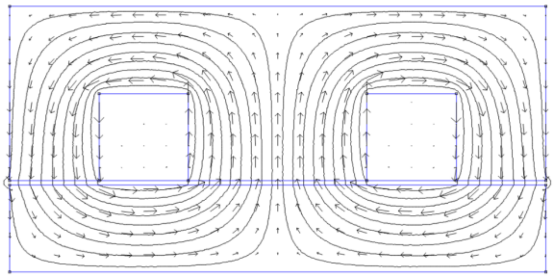

Figure 20. Equipotential lines and magnetic flux density vectors

Table 2. Force vs airgap

Force [kN]	Air gap [mm]
10.0966	1
2.62523	2
1.19401	3
0.681467	4
0.439802	5
0.306505	6
0.225328	7
0.172021	8
0.135105	9
0.108462	10

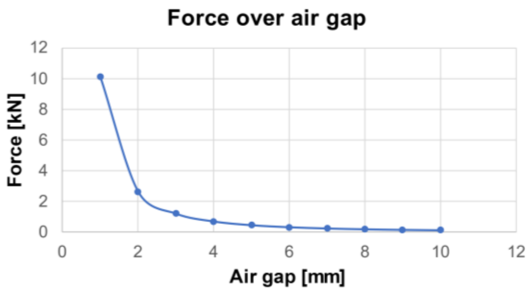

Figure 21. Force variation depending on air gap

4.2.2. Case 2: Fixed Air Gap, Variable Current

In this case, a constant air gap of 1 mm was considered, and the current varies from 1 to 10 A. Table 3 contains the values of the magnetic force for each value of the current. The force variation with the current is seen in Figure 22, [40-42].

Table 3. Force vs current

Force [kN]	Current [A]
2.52423	1
10.0969	2
22.718	3
40.3876	4
63.1057	5
90.8721	6
123.687	7
161.55	8
204.462	9
252.423	10

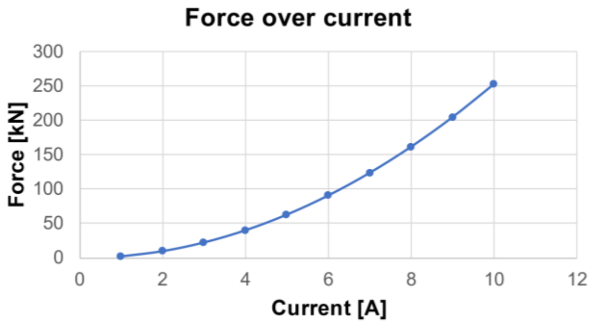

Figure 22. Force variation depending on current

5. CONCLUSIONS

This paper presents some application for electro – mechanical fields related to the transformation of electromagnetic energy into mechanical energy. Electromagnetic phenomena analysis takes into account the side effects of associated phenomena, such as mechanical, chemical, optical or thermal. In engineering practice, an electromagnetic field analysis problem consists in determining the electromagnetic field corresponding to given conditions of uniqueness Therefore, the mechanical forces and moments or the ponderomotive interactions exerted on the bodies in a closed domain together with the electromagnetic field prove the transformation of the electromagnetic energy into mechanical energy respecting the law of energy conservation. If the total energy of the electromagnetic field is known, its interaction with any physical system consisting of bodies can be described by means of electromagnetic forces. The additive separation of the electromagnetic force, in volume density of the electric and magnetic force is used as a basis of the applications developed in this paper.

The readers are familiarized with theoretical notions related to quantities and of the general laws of the macroscopic theory of the electromagnetic. Then, the electromagnetic energy theorem is stated and proved. Many engineering applications imply the calculation of force generated by the electric field of a plate capacitor or the carrying force of an electromagnet these examples being found in many electromechanical equipment. Therefore, a special Section is dedicated to briefly describe the constructive and functional characteristics of some equipment.

The last Section is dedicated to numerical calculation methods of the electromagnetic field and of the electromagnetic energy. The FEMM programming environment has all the modules necessary to use the finite element method, for any linear and nonlinear structures. There are analyzed three applications of numerical computation of electromagnetic forces using the results of simulations performed in FEMM software environment. These computational examples were chosen from real devices that are used in complex systems to generate electromagnetic forces dependent on mechanical displacements or variable electrical quantities. The dependence between the electromagnetic and the mechanical field is synthesized, through the direct analysis of the forces generated by the electromagnetic field.

NOMENCLATURES

1. Acronyms

FEM Finite element Method
FEMM Finite Element Method Magnetics
emf Electromotive force
mmf Magnetomotive force

2. Symbols / Parameters

Ψ_Σ : Electric flux
q_{V_Σ} : Electric charge
Φ : Magnetic flux
$\bar{D}$: Electric flux density
$\bar{B}$: Magnetic flux density
ε: Absolute permittivity of the medium
μ: Absolute permeability of the medium
σ : Conductivity
ρ_V : Charge volume density
$\bar{A}$: Magnetic vector potential
u_Γ : Electromotive force
u_{m_Γ} : Magnetomotive force
(Γ) : Closed curve
(Σ) : Closed surface
(V_Σ) : Domain bounded by (Σ)
$d\bar{l}$: Line element
$d\bar{A}$: Area element
i_{S_Γ} : Conduction current
i_{dS_Γ} : Displacement current
$i_{v_{S_\Gamma}}$: Convection current
$i_{R_{S_\Gamma}}$: Roentgen current
dW: Variation of electromagnetic energy
P_Σ : Transmitted electromagnetic power inside the considered domain
P_t : Power transformed into other forms of energy
$\bar{S}$: Poynting vector
w: Volume density of the electromagnetic energy
$\bar{f}$: Force density
$\bar{f}_e$: Volume density of the electric force
$\bar{f}_m$: Volume density of magnetic force
W_m: Magnetic energy
W_e: Electric energy
L: Inductance
M: Mutual inductance
X_k: Generalized force
R_m: Reluctance
N: Number of coils turns
V: Electric potential
C: Capacitance
U: Voltage

REFERENCES

[1] B.W. Jaekel, "Electromagnetic Environments - Phenomena, Classification, Compatibility and Immunity Levels", Int. Conf. IEEE-EUROCON, pp. 1498-1502, 2009.
[2] B.W. Jaekel, "Description and Classification of Electromagnetic Environments - Revision of IEC 61000-2-5", IEEE International Symposium on Electromagnetic Compatibility, pp. 1-4, 2008.
[3] V.I. Moshkin, D.N. Shestakov, G.G. Ugarov, "Energy Modes of the Pulse Linear Electromagnetic Motor with Effective Transformation of Magnetic Energy", International Multi-Conference on Industrial Engineering and Modern Technologies (FarEastCon), pp. 1-3, Vladivostok, Russia, 2018.
[4] G. Ochiana, L. Ochiana, M. Stanculescu, "Electromagnetic Field Theory", PrinTech Press, Bucharest, Romania, 2012.
[5] R.C. Dorf, J.A. Svoboda, "Introduction to Electric Circuits", Wiley, 8th Edition, New York, USA, 2010.
[6] W. Hayt, J. Kemmerly, S. Durbin, "Engineering Circuit Analysis", McGraw-Hill, 8th Edition, New York, USA, 2011.
[7] D.K. Cheng. "Field and Wave Electromagnetics", Addison-Wesley, 2th Edition, New York USA, 1989.
[8] J.W. Nilsson, S. Riedel, "Electric Circuits", Prentice Hall, Edi. 9, New York, USA, 2010.
[9] J.D. Kraus, "Electromagnetics with Applications", McGraw-Hill, 5th Edition, New York, USA, 1999.
[10] M.O. Sadiku, "Elements of Electromagnetics", Oxford University Press, 5th Edition, London, England, 2009.
[11] C. Alexander, M.O. Sadiku, "Fundamentals of Electric Circuits", McGraw-Hill, The 5th Edition, New York, USA, 2012.
[12] N. Ida, "Engineering Electromagnetics", Springer, 2nd Edition, Amsterdam, Netherlands, 2007.
[13] E. Cazacu, I.V. Nemoianu, "Special Magnetic Devices; Computation and Theory Elements", Matrix Rom Press, Bucharest, Romania, 2008 (in Romanian).
[14] M. Stanculescu, P. Andrei, H. Andrei, S. Deleanu, L. Bobaru, "Numerical Assessment of Electromagnetic Energy and Forces in Non-Destructive Measurement Devices", Chapter in: "Numerical Methods for Energy Applications", N.M. Tabatabaei, N. Bizon (Eds.), Springer, Cham, Switzerland, 2021.
[15] H. Andrei, P. Andrei, L.M. Constantinescu, R. Beloiu, E. Cazacu, M. Stanculescu, "Electrical Power Systems",

Chapter in: "Reactive Power Control in AC Power Systems", N.M. Tabatabaei, N. Bizon (Eds.), Springer Cham, Geneve, Switzerland, 2017.
[16] H. Andrei, F. Stan, "Modern Electrical Engineering. Electrical Engineering and Electromechanical Converters. Theory and applications", Bibliotheca Press, Vol. 1-2, Targovishte, Romania, 2010 (in Romanian).
[17] N.F.O. Serteller, G. Kari, "Understanding the Foundations of Electromagnetic Field Theory with Computer Software", IEEE Int. Conf. on Global Engineering Education Conference (EDUCON), pp. 755-760, 2019.
[18] N.N. Duan, W.J. Xu, S.H. Wang, H.L. Li, Y.G. Guo, J.G. Zhu, "Extended Finite Element Method for Electromagnetic Fields", IEEE Int. Conf. on Applied Superconductivity and Electromagnetic Devices (ASEMD), pp. 364-365, 2015.
[19] Y. Zhao, T. Wen, Y. Li, H. Ni, Q. Zhang, W. Chen, "A FEM-Based Simulation of Electromagnetic Forces on Transformer Windings Under Short-Circuit", IEEE Int. Conf. on Power Modulator and High Voltage (IPMHVC), pp. 425-429, 2018.
[20] L. Qiu, "Analysis of Electromagnetic Force and Deformation Behavior in Electromagnetic Tube Expansion with Concave Coil Based on Finite Element Method", IEEE Transactions on Applied Superconductivity, Vol. 28, No. 3, pp. 1-5, April 2018.
[21] Z. Liu, J. Wang, M. Wang, Y. Lu, R. Huang, Y. Chen, "Finite Element Method and Charge Coupling Method for Distortion Field of AC Transmission Line", IEEE Int. Conf. on Sustainable Power and Energy (iSPEC), pp. 1049-1053, 2019.
[22] M. Demenko, J. K. Sykulski, "Analogies Between Finite-Difference and Finite-Element Methods for Scalar and Vector Potential Formulations in Magnetic Field Calculations", IEEE Transactions on Magnetics, Vol. 52, No. 6, pp. 1-6, June 2016.
[23] X. Wu, F. Grassi, P. Manfredi, D. Vande Ginste, "Efficiency of the Perturbative Stochastic Galerkin Method for Multiple Differential PCB Lines", IEEE-Int. Conf. Electrical Design of Advanced Packaging and Systems Symposium (EDAPS), pp. 1-3, 2018.
[24] Z. Peng, B. Mackie Mason, "Integral Equation discontinuous Galerkin methods for time harmonic Electromagnetic Wave Problems", Int. Review of Progress in Applied Computational Electromagnetics (ACES), pp. 1-2, 2015.
[25] L. Hu, K. Yang, C. Suo, Y. Ding, W. Yu, "Analysis of Radial Electromagnetic Force in Permanent Magnet Machine with Manufacturing Tolerance", Int. Conf. on Electrical Machines and Systems (ICEMS), pp. 256-260, 2018.
[26] L. Qiu, "Electromagnetic Force Distribution and Wall Thickness Reduction of Three-Coil Electromagnetic Tube Bulging with Axial Compression", IEEE Access, Vol. 8, pp. 21665-21675, 2020.
[27] E. Scholz, S. Lange, T. Eibert, "Electromagnetic Field Theory as System Theory", Int. Symposium on Electromagnetic Theory (EMTS), pp. 733-736, 2016.
[28] B. Sanosian, P. Wendling, T. Pham, W. Akaishi, "Electromagnetic Forces on Coils and Bars Inside the Slot of Hydro-Generator", IEEE-Energy Conversion Congress and Exposition (ECCE), pp. 1754-1760, 2019.
[29] T. Dai, H. Li, J. Li, B. Yuan, X. Liu, "A Hybrid Calculation Method of Radial Electromagnetic Force Based on Finite Element Method and Analytic Method in A Permanent Magnet Synchronous Machine", IEEE Int. Conference on Electrical Machines and Systems (ICEMS), pp. 1-4, 2019.
[30] Y. Zhao, T. Wen, Y. Li, H. Ni, Q. Zhang, W. Chen, "A FEM-based Simulation of Electromagnetic Forces on Transformer Windings Under Short-Circuit", IEEE Int. Conf. on Power Modulator and High Voltage (IPMHVC), pp. 425-429, 2018.
[31] Y. Boutora, N. Takorabet, "A Novel Direct Resolution Method for Coupled Systems in Finite Element Analysis", Int. Journal For Computation And Mathematics In Electrical And Electronic Engineering (COMPEL), Vol. 39, Issue 5, pp. 1271-1280, 2020.
[32] W. Schoenmaker, P. Galy, "Integrated Front-End/Back-End Simulation of Electromagnetic Fields, Lorentz Force Effects and Fast Current Surges in Microelectronic Protection Devices", Int. Conf. on IC Design & Technology (ICICDT), pp. 1-4, Leuven, Belgium, 2015.
[33] Y. Zhang, C. Zhuang, R. Zeng, "Electrical Field Evaluation Around Slender Conductors by Collocation Boundary Element Method", IEEE Int. Conf. on Electromagnetic Field Computation (CEFC), pp. 1-7, 2016.
[34] Y. Xu, H. Fang, D. Li, R. Qu, J. Guo, "Fast Evaluation of High Frequency Electromagnetic Force and Vibration for Electrical Machines Based on Field Reconstruction Technique", IEEE Int. Conf. on Electric Machines and Drives (IEMDC), pp. 1849-1854, 2019.
[35] W. Wang, H. Zhang, "The Analysis of Magnetic Field and Electromagnetic Force for Transverse Flux Permanent Motor", Chinese Automation Congress (CAC), pp. 5890-5895, 2020.
[36] M. Popa, A. Dolan, "Static Force Characteristic of e-Type Single Phase AC electromagnets", Int. Conf. on Applied and Theoretical Electricity (ICATE), pp. 1-5, 2012.
[37] C.S. Wang, B.Y. Duan, "Electromechanical Coupling Model of Electronic Equipment and its Applications", IEEE Int. Conf. on Mechatronics and Automation, pp. 997-1003, 2010.
[38] M. Stanculescu, M. Maricaru, I.F.Hantila, S. Marinescu, L. Bandici, "An Iterative Finite Element - Boundary Element Method for Efficient Magnetic Field Computation in Transformers", Rev. Romaine Sci. Tech. Electrotechnics and Energy, Vol. 56, Issue 3, pp. 267-276, 2011.
[39] M. Maricaru, P. Minciunescu, P. Andrei, W. Kappel, C. Grumeza, I.F. Hantila, "Evaluation of the Ferromagnetic Materials B-H Static Characteristic", Rev. Romaine Sci. Tech. Electrotechnics and Energy, Vol. 63, Issue 3, pp. 249-252, 2018.

[40] P. Andrei, M. Maricaru, S. Marinescu, M. Stanculescu, F.I. Hantila, "An Efficient Procedure to Assess the Static Magnetization Relationship", Rev. Romaine Sci. Tech. Electrotechnics and Energy, Vol. 61, Issue 2, pp. 101-105, 2016.
[41] P. Andrei, "Device for Determining the First Magnetization Characteristic for Ferromagnetic Materials", PhD. Thesis, Polytechnics University of Bucharest, Romania, 2015.
[42] H. Andrei, C. Fluerasu, E. Virjoghe, Co. Fluerasu, D. Enescu, D. Popovici, Adela Husu, P. Andrei, G. Predusca, E. Diaconu, "Numerical Methods, Modelling and Simulation in Electrical Engineering", Electra Press, Bucharest, Romania, 2011 (in Romanian).

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