

EXPERIMENTAL RESEARCH OF THE CYLINDRICAL SHELLS STABILITY WITH RECTANGULAR HOLES OF DIFFERENT SIZES UNDER TRANSVERSAL BENDING

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Abstract- The results of a systematic experimental study of the subcritical behavior, supercritical forms of equilibrium and critical loads of cylindrical shells with a rectangular cutout of the lateral surface during transverse bending are presented. The hole was located in the compressed zone of the shell. Five series of models (243 shells) were tested, for which the area and aspect ratio of the rectangular hole varied within wide limits. Models for research were made from special drawing paper, which has stable mechanical characteristics and high manufacturability of models. It is shown that despite the relatively low values of the elastic characteristics of this material in comparison with metals, the ratio of its yield strength to the modulus of elasticity is commensurate with these data for sheet stainless steel, which makes it possible to carry out large-scale experimental studies that are practically impossible to carry out on metal shells. It is indicated that the loss of stability of a cylindrical shell under conditions of transverse bending has its own characteristics due to the inhomogeneous stress state of the shell, both along the length and in the circumferential direction. In the zones of maximum normal compressive stresses, the forms of buckling are similar to the forms of buckling of the longitudinally compressed shell, and in the zones of maximum shear stresses, they are similar to the forms of buckling during torsion. The article presents and analyzes the test data of shells for holes of various areas and configurations (aspect ratios) of a rectangular hole elongated along the generatrix, in the circumferential direction and square. The results are presented in the form of pictures of buckling forms and graphs of the dependence of the critical transverse force on the aspect ratio for different areas of the hole.

Keywords: Cylindrical Shell, Rectangular Hole, Stability, Transverse Bending, Experimental Study.

1. INTRODUCTION

Thin-walled cylindrical shells, which, according to design and technological requirements, often have holes (hatches) of various shapes and sizes, are widely used in rocket and space, aviation, oil refining, chemical and other industries.

The presence of a hole in a smooth cylindrical shell leads to a significant change in the stress-strain state. Along with the main stress state, which arises in a shell without holes under the action of an external load, areas of local stress concentration arise around the holes. The location of such areas in the compressed zone of the shell often initiates first a local loss of stability with the formation of large deformations and a significant change in the stress field, and then a general loss of the bearing capacity of such a structural element. The change in the stress-strain state during loading, the appearance of forms (change in the surface geometry) of local and further general loss of stability are essentially nonlinear. At the same time, a reliable theoretical (numerical) determination of the value of the critical (breaking) load corresponding to the general loss of stability of the shell is often very difficult. This is due not only to the complexity of solving the corresponding nonlinear problem, but also to the fact that the calculations take into account the "idealized" input parameters of the shell with holes (initial imperfections in shape, size, properties, etc.), which cannot be provided in any the most perfect experiment, and even less so in a real design.

Under these conditions, it is extremely important to carry out correctly staged experimental studies, which is associated, however, with significant logistical difficulties. These circumstances determine the relevance of experimental studies of this problem, since ensuring the stability and bearing capacity of such thin-walled shells under the action of various external loads on them is one of the most important engineering problems.

One of the first articles devoted to experimental studies of the effect of holes on the stability of shells was the work of R. Tennyson [1], who studied longitudinally compressed cylindrical shells weakened in the middle by one circular hole of various diameters. The casings were made of epoxy by centrifugal casting. The work [2] presents the results of an experimental study of the stability of circular longitudinally compressed cylindrical shells, weakened in the middle by one circular cut under the action of axial compression, torsion, external pressure and a combination of these loads. The experiments were carried out on shells made of a triacetate film.

In the works of D.G. Starnes [3], the results of an experimental study of the effect of circular and rectangular cuts on the stability of thin cylindrical shells made of mylar polystyrene film under the action of axial compressive force, bending and torsion are discussed.

In the monograph by V.I. Mossakovskiy, et al. [4] give a number of test results for the stability of shells made of sheet steel. For the first time, the possibility of studying thin-walled shells made of special drawing paper was shown by R.V. Rhode, E.E. Lundquist [5], and later V.M. Chebanov [6] and E.F. Prokopalo [7]. To date, there are a number of works devoted to the study of the stability of thin-walled shells under the action of various types of loading, among which the following should be noted [8-18].

Known results of experimental researches represent test data of only a relatively small quantity of shells models, taking into account their high material and technical costs. The shells were made from different materials and sizes, covered only a small range of hole parameters and were using different experimental technologies. Generalization and carrying out quantitative comparisons of the obtained results, taking into account the presence of a scale factor widely known in mechanics, is very problematic.

The advantage of the results presented in the article is the use of cylindrical shell models of high identity and manufacturability. This made it possible to carry out systematic researches of a large number of models with a change in the studied parameter under the same.

2. MODELS FOR TESTING

In this work, GOST 597-73 "B" grade drawing paper was chosen as a material for making models. As the results of additional experimental studies have shown, this material has fairly stable mechanical characteristics, and most importantly, high manufacturability in the manufacture of models. The substantiation of the correctness of the use of such drawing paper for the manufacture of models of cylindrical shells is given in [5, 6, 7, 11, 19], where it is shown that, despite the orthotropy of mechanical properties and the relatively low values of the elastic characteristics of paper in comparison with metals, the ratio of the yield strength of this material to the modulus elasticity (σ_T/E) comparable to these data for stainless steel sheet.

This parameter is an important indicator of the quality of the research carried out, since the natural desire of the experimenter to study the stability of the shell before the loss of its strength, that is, in the elastic region, requires a decrease in its relative thickness δ/R (where δ is the wall thickness, and R is the radius of curvature of the shell), which leads to an increase in the influence (in comparison with δ) of the initial geometric imperfections and, as a consequence, to the leveling of the actual effect of a change in the investigated parameter on the shaping and quantitative parameters of stability loss.

On the other hand, the desire to reduce the influence of such initial geometric imperfections of the shape by increasing the parameter δ/R lead to the appearance of

plastic deformations, even before the loss of stability of the object of research. Based on this, the use of paper makes it possible to carry out large-scale experimental research, which is practically impossible to perform on metal shells.

As a result of preliminary studies carried out both on flat samples and on shells using an electromechanical system, the main mechanical characteristics of the material were determined: elastic moduli $E_x = 6.9 \times 10^9$ Pa, $E_y = 3.45 \times 10^9$ Pa, $G = 1.92 \times 10^9$ Pa, Poisson's coefficients $\mu_x = 0.3$, $\mu_y = 0.15$. Here indices X , Y correspond to the main directions of paper orthotropy. The sheet thickness was $h = 0.23 \times 10^{-3}$ m. This value was practically unchanged for the corresponding batch of paper. Measurement of the sheet thickness was carried out using a dial indicator with a scale division of 10^{-6} m.

For the manufacture of shells from a sheet of «Whatman» paper, rectangular blanks were cut so that their sides were parallel to the main directions of orthotropy. The E_x -direction always coincided with the direction of the forming shell, and E_y with its direction. A rectangular hole was cut out on the blanks using a special tool and templates, the contour lines of which coincided with the lines of curvature of the shell and which was located in the compressed zone diametrically opposite to the glue seam symmetrically relative to the middle cross section of the shell. The plane of action of the transverse force in all tests passed through the middle of the glue joint, which, therefore, was in the region of tensile stresses.

The developed technology ensured the absence of visually noticeable permanent deformations in the area of the holes. Then the workpiece was glued on a metal cylinder with BF-2 glue. The width of the glue line was 5×10^{-3} m. The inner radius of all models was $R = 37.5 \times 10^{-3}$ m, working length $L = 75 \times 10^{-3}$ m. The total length of the shell was taken as $L_F = 115 \times 10^{-3}$ m. To ensure the possibility of uniform transfer of axial compressive forces to the shell, metal end devices with a circular cross-section with a diameter of $d = 75 \times 10^{-3}$ m were glued to its curved edges. The width of the glue was 2×10^{-2} m. Such anchoring of the edges provided boundary conditions close to rigid pinching.

To research the influence and size of the rectangular hole and the configuration (aspect ratio) on the stability of the cantilever cylindrical shell during transverse bending, 5 series of models (243 pieces) of shells were manufactured and tested. For each series, the area of the F holes was as follows:

I series - $F = 4 \times 10^{-4}$ m²;

II series - $F = 8 \times 10^{-4}$ m²;

III series - $F = 12 \times 10^{-4}$ m²;

IV series - $F = 16 \times 10^{-4}$ m²;

V series - $F = 20 \times 10^{-4}$ m², and within the series constant.

The change in the ratio of the sides (geometric dimensions) of rectangular holes for the shells of the tested series varied widely: from $a/b = 0.04$ to $a/b = 10$, where a , b are the sides of the rectangular hole along the arc of the shell and its generator, respectively.

Tests of such cylindrical shells on the action of transverse force were carried out using a special device, the schematic diagram of which is shown in Figure 1. The shell (1) with the lower end (2) was fixed to the massive horizontal plate (3). The load was carried out through a bracket (4) attached to the upper end (5). The bracket had a hole in which a flexible cable (7) was fixed, which passed further through the block (8) and served to transmit force from the loads (6) to the shell. Due to the block, no friction was achieved, as a result of which the forces in the horizontal and vertical directions of the cable were the same. After the shell was mounted and the bracket was installed in the desired position, with the help of standard weights (6) of the second accuracy class (6) the step loading of the shell began to lose stability.

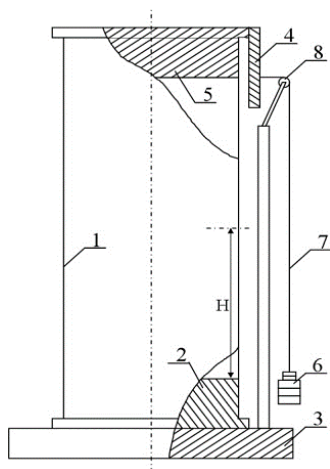


Figure 1. Scheme of the transverse load of the shell with a hole

3. RESEARCH RESULTS

It should be noted that in the conditions of transverse bending the deformation of the shell has its own characteristics. In it there is an inhomogeneous stress state associated with the presence of both tangential and normal stresses, which are inhomogeneous in both length and arc of the shell. The corresponding form of loss of stability is determined by the ratio between the maximum values of these stresses and depends on the geometric parameters of the shell and the hole, as well as on (coordinates) the location of the zone on the surface.

It is known [4, 10] that when normal stresses predominate in the shell, the form of buckling is the same as in longitudinal compression, and in the case when shear stresses play a decisive role, the form of buckling is similar to torsion. This feature is well evidenced by the experimentally obtained forms of buckling of continuous shells.

In the zone of maximum normal compressive stresses, as a result of buckling, diamond-shaped dents were formed, which are characteristic of the buckling of a cylindrical shell under longitudinal compression. In the zones of maximum tangential stresses, elongated dents inclined to the shell axis were characteristic of shell buckling during torsion.

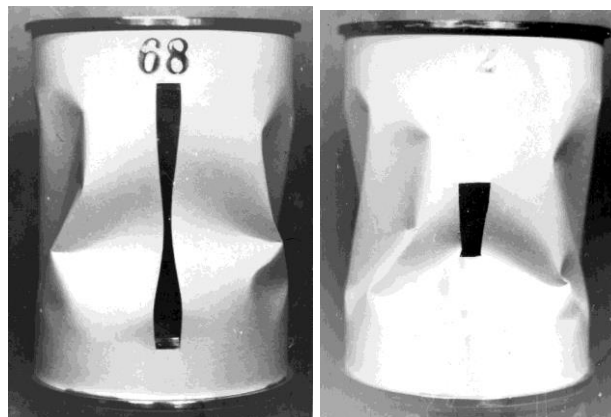


Figure 2. Forms of loss of stability of a cylindrical shell with a longitudinally elongated along the axis ($a/b < 1$) rectangular hole with transverse bending

When testing shells with unsupported rectangular holes with different aspect ratios, the following main features of the behavior of the shells were identified:

For the case of shells with a rectangular hole placed in a compressed zone, elongated along the forming shell ($a/b = 0.2-1.0$), with increasing load, the larger sides of the hole were first bent outwards. With further loading, this curvature increased, and then these sides of the contour bent sharply inside the shell and there was a general loss of stability, which was accompanied by clap. The surface of the shell was covered with additional dents (Figure 2).

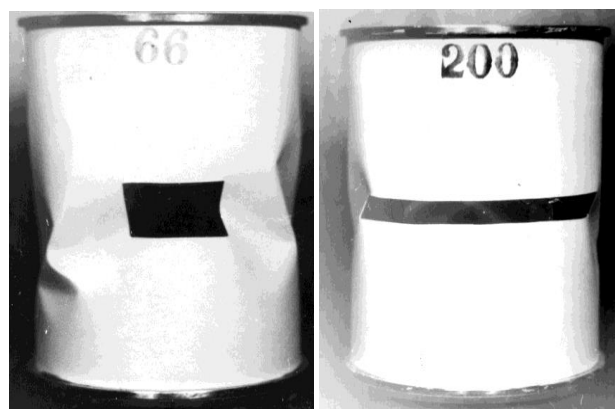


Figure 3. Forms of loss of stability of a cylindrical shell with a rectangular hole, elongated in the circumferential direction ($a/b > 1$) with transverse bending

In the case (Figure 3), when the hole was extended along the arc of the shell ($a/b = 1.1-10$), with increasing load in the area of the hole there was a barely visible increase in bending deformations. Then local dents formed in the area of the hole.

In this case, the local loss of stability of the shell near the edges of the hole occurred due to the development of precritical bending deformations. With further loading, the area of the dents increased slightly, their nodal lines gained a slope and then there was a general loss of stability with the formation of a series of bulges. With the loss of stability of the shell with square holes with increasing load at first near the holes (in the corner zones)

smoothly, or accompanied by "soft" clap, local dents appeared, which further developed and at some point, there was a general loss of stability of the shell, accompanied by a "sharp" clap with the formation of diamond-shaped dents characteristic of axial compression, which indicated that normal stresses were predominant in this case Figure 4(a). In the lateral zones, where the maximum tangential stresses took place, inclined waves were formed Figure 4(b), resembling the forms of loss of torsion stability.

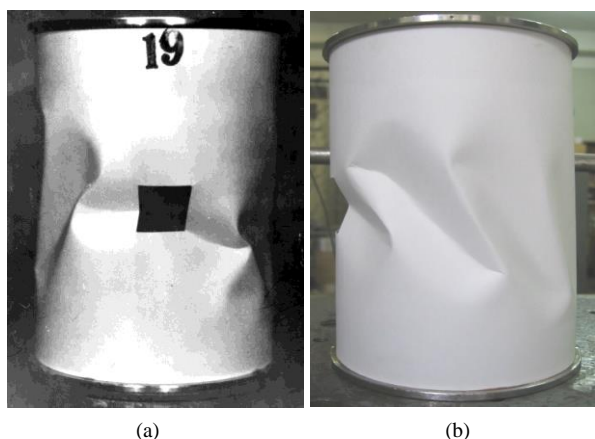


Figure 4. Typical forms of loss of stability of a cylindrical shell with square ($a/b=1$) holes in transverse bending

The nature of the supercritical equilibrium forms for the tested shells was as follows. When testing continuous shells, 11 dents were formed on their surface. For hole shells, the number of dents decreased with increasing hole area. Thus, for the case when the hole was quite significant ($a/b = 0.2-0.5$), stretched along the forming number of dents for the shells of the 1st series was 10 dents, for the 2nd and 3rd series, 9 dents, for the 4th and 5th series, 8 dents.

When the ratio of the sides was $a/b = 0.6-1.67$, the loss of stability of the shells of the 1st series occurred, as a rule, with the formation of 8 dents, the 2nd and 3rd series of the 7th dents, 4th and 5th series of 6 dents. When the hole was extended in the transverse direction of the shell ($a/b = 2.5-10$), the dents grew faster, but their number was smaller. Thus, for the 1st and 2nd series, loss of stability occurred with the formation of 6 dents, for the 3rd and 4th series, 5 dents, and for the 5th series, 4 dents.

In Figure 5 presents the results of tests of shells in the form of the dependence of the relative critical load $\bar{Q}_{cri} = Q_{cri}^o / Q_{cri}$ on the ratio a/b of the sides of a rectangular hole for shells with different hole sizes, where the symbol „o” indicates data for shells with hole $F = 4 \times 10^{-4} \text{ m}^2$, „●” – $F = 8 \times 10^{-4} \text{ m}^2$, „Δ” – $F = 12 \times 10^{-4} \text{ m}^2$, „■” – $F = 16 \times 10^{-4} \text{ m}^2$, „□” – $F = 20 \times 10^{-4} \text{ m}^2$, respectively, for series I-V.

Here, Q_{cri}^o, Q_{cri} the critical forces of loss of stability of the shell without a hole and in the presence of a hole, respectively $Q_{cri} = 112 \text{ N}$, and the total area of the side surface of the shell is $314 \times 10^{-4} \text{ m}^2$. All dependencies have a qualitatively similar nature of change.

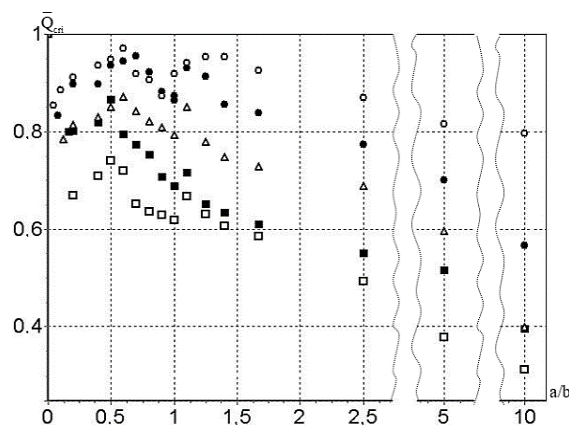


Figure 5. Dependence of the reduced critical transverse force \bar{Q}_{cri} on the shape of a rectangular hole of different area (Series I-V)

The obtained results are characterized by a general tendency to decrease the magnitude of the critical loads with increasing hole area and with a change in its configuration (increasing a/b). It is also noteworthy that for the shells of all series, the critical loads reach their maximum values for the ratio of the sides of the hole $a/b = 0.6$ and $a/b = 1.1$ (Figure 5). In this case, for example, for shells of the first series, the presence of a hole $F = 4 \times 10^{-4} \text{ m}^2$ for this ratio of the parties leads to a decrease in bearing capacity by only 2-4 % compared to a solid shell.

In the area $a/b = 0.6-1.1$ there is a slight (about 9 %) decrease in the critical load, which is achieved at $a/b = 1$, i.e., for the case of a square hole. This, obtained for all tested series, the non-uniformity of depends the critical load on the ratio of the sides of the hole in this range of the ratio a/b can obviously be explained by the above "game" of normal and tangential stresses, also due to the presence of a stress concentrator and, as a consequence, wave formation around the hole.

The experimental results of tests of cylindrical shells with one rectangular hole of various areas and configurations presented in the article in the form of values of the critical buckling force \bar{Q}_{cri} and waveforms can be useful for testing software tools for numerical analysis of the behavior of such structural elements. The point is that, the considered stability problems are essentially geometrically nonlinear. Large displacements of the shell surface develop in the process of loss of bearing capacity, initially in the form of a local and then a general loss of stability. This problem is even more complicated due to the presence of hole as stress concentrator. In this case, the zones of convexity (concavity) of the surface of wave formation in the process of transition from local to general loss of stability can change places (this is the phenomenon of branching of solutions of nonlinear equations). Such a case of loss of stability of the shell was observed in the experiments carried out in the presence of holes $a/b = 0.2 \div 1.0$. The investigation of such problems by numerical simulation methods requires great engineering skill.

4. CONCLUSIONS

The article presents the results of experimental studies of the influence of a rectangular hole of different area (5 series) located in the compressed zone and the aspect ratio on the critical force of loss of stability of cylindrical shells under the action of transverse bending.

Estimation of scatter in tests of identical models testifies to high stability of experimental results. Most scatter did not exceed 3-5%.

It is established that the number of waves decreases with increasing hole area and in the case when the hole is extended along the arc of the shell.

It is established that the appearance on the shell of a narrow longitudinal hole ($a/b = 0.04$) of different areas reduces the bearing capacity by 12-35%, and the appearance of a narrow transverse hole ($a/b = 10$) for the same values of areas - already in range from 20% to 70%. Thus, the appearance of a cross-section (or crack) in the shell is significantly more dangerous for the loss of bearing capacity of the shell than the same section (or crack) in the longitudinal direction. The results obtained can be useful in the design of structures containing cylindrical shells with holes, as well as in predicting the residual bearing capacity of such structural elements that have received damage.

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