International Journal on "Technical and Physical Problems of Engineering" (IJTPE)
Published by International Organization of IOTPE
ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com
September 2022 | Issue 52 | Volume 14 | Number 3 | Pages 133-143

PROBLEM SOLVING TECHNIQUES OF ENGINEERING APPLICATIONS

M. Hedayati [1] R. Effatnejad [1] K. Choopani [1] F. Koneshloo [2] M. Effatnejad [3] A. Effatnejad [4]

1. Department of Electrical Engineering, Karaj Branch, Islamic Azad University, Karaj, Iran
m.hedayati@kiau.ac.ir, reza.efatnejad@kiau.ac.ir, chopani@jku.ac.ir
2. Department of Electrical Engineering, University of Ghiaseddin Jamshid Kashani, Qazvin, Iran
fatemeh_koneshloo@yahoo.com
3. Department of Management and Economic, Science and Research Branch, Islamic Azad University, Tehran, Iran
mohammadeffatnejad1375@gmail.com
4. Department of Medical, Shahrood Branch, Islamic Azad University, Shahrood, Iran, aliaen@yahoo.com

Abstract- There are several approaches to optimize engineering models. This chapter proposes the two methods of Lagrange release and convex optimization to solve engineering problems. In the Lagrange release algorithm, the constraints specify that are more difficult to satisfy with a specific coefficient to the objective function of the sample and its value with the help of sub-gradient methods in different iterations. Lagrange release algorithm is one of the most widely utilized innovative methods in calculating mathematical constrained optimization models. This algorithm, which is developed based on Lagrange Case to solve constrained optimization problems, frees all or some limitations of the problem while providing information from the optimal solution of the main problem, creating acceptable approximate answers for the main problem, which are mainly from these solutions can be used as a boundary in other optimization algorithms. But another way to solve problems is a convex optimization. Convex optimization is the only method that can guarantee a globally optimal solution in engineering models. Because in convex modeling, there is only one optimal solution. There are several ways to solve convex optimization problems. We present the convex mathematical models for solving engineering problems. We then implement these models in the GAMS Software and analyze the results. We apply the convex and non-convex models of Economic Dispatch (ED) subject to the software and analyze the results of these two cases. So, in this chapter, we present two methods for solving nonlinear problems. In the first method, we solve the nonlinear model by releasing Lagrange. In the second method, we linearize and compare the results with each other.

Keywords: Convex Optimization, Lagrange Release, Economic Dispatch, Non-Linear Models.

1. INTRODUCTION

ED is a significant problem for ISO / RTOs to earn the ED exploitation of the power system in the markets of day-ahead and real-time. This goal is usually for optimally the entire cost of generation while meeting the system schedule and operational requirements. Conventional ED is done in real-time every 5 minutes for a moment that often is called Static Economic Dispatch (SED). Also, dynamic ED (DED) is used to improve the instantaneous ED to a multistage problem. DED is used to forecasted load for a special interval considering power plant ramp speed limitations.

A neural network (NN) utilizing multiple processors is considered in [1] to study the DED problem. Also, the DED problem is utilized in unregulated systems [2]. A NN utilizing multiple processors is considered in [1] to survey the DED problem. Also, the DED model is used in disordered systems [2]. Recently, as a replacement for the conventional SED, prospective ED has converted a modern industry standard, which is a DED subject, with alone first-stage ED decisions being implemented. The updated forecast information is used in the ED problem for future decisions. Thus, DED is crucial for operational operation and dispatch before planning is generated so that the system can properly control sudden variations in load soon. The DED problem by a large number of intervals is a very large spatial-temporal planning subject that may pose a major computational problem.

In general, there are many methods for solving DED, include dynamic programming, genetic methods, simulated annealing method, and other heuristic approaches. However, the selection of parameters significantly affects the efficiency of approaches. With the improvement of mathematical planning, simple methods and interior points are used due to their robustness in the fields of industrial research and engineering. To improve computational performance, advanced decomposition models are extensively checked in past years to divide the main very large model to - sub-models when are solved in parallel. In addition, the Lagrange relaxation (LR) approach while is equal for a two-step plan, is considered in [3] to divide a very large optimization subject by associated nature to multiple small sub-problems.

In [4], the step capacity of the dual ascent approach to calculate the dual problem is determined properly and multiple other presumptions are maintained, this approach can be converged. Ref. [5] used the Gaussian-Newton approach to update the coefficients, which reported proper solutions. Although, the Lagrange relaxation approach considered a great number of coefficients for the relaxation link limitations, which raises the challenges of the Lagrange dual model. However, the Quasi-Newton approach for storing and refreshing the Hessian matrix requires a large amount of memory space, which can take a lot of CPU time. In addition, [6] reported that Lagrange relaxation hypotheses are generally robust in some operating applications. In [21], the most used Evolutionary Computation based models and their applications on OPF are reviewed and also some unused Evolutionary Computation based models for OPF are also presented. In [22], tried for demonstrate reactive power optimization model, miscellaneous targets, voltage stability indexes types and formulization of them, reviewing recent studies in this filed and comparison between them for studding efficiency of them.

To solve the mentioned models, the augmented LR (ALR) approach is strengthened, to some extent, the robustness for multiple updates, the experience of simplicity and stability of coefficient repetitions, and especially, for convergence without assumptions exact include convexity or finiteness of cost function was developed. It is due to the ALR considers an additional second-class penalty, the advantage of which is that in relatively mild conditions it can distinguish the dual problem from the main model. Unfortunately, the contributed penalty in the ALR function is non-derivative and leads to the problem of direct decomposition compared to the LR approach. This difference is critical for the ALR method. For the most common case, the alternating direction method of multipliers (ADMM) model combines dual ascent decomposition with stronger convergence. In ADMM, the common element of variables is updated periodically or sequentially, where a Gauss-Seidel pass is used over common variables.

Recently, [7] has considered a diagonal second-class estimation approach for approximating the duration of non-derivative penalties. The optimal solution for convex optimization is also proved. Another problem by both LR and ALR is that limitations are evermore divided into "easy limitations" that can be maintained and "hard limitations" that can be maintained with cost functions. So, choosing "hard constraints" is essential. LR is commonly used for "hard constraints" such as energy balance and security limits. However, for the following reasons, we will choose ramp limits as "hard limitations":

1. ramp rate limitations for parallel computation have a much better degree of derivation from relaxing energy equality limitations and safety limitations, meaning that the relaxing ramp rate limitations can lead to better performance.
2. In power networks by many fast-ramping gas-fired generators, relaxing ramp rate limits have more advantages from systems with slow-ramping coal generators.
3. Today, in the operational exploitation and dispatch of the power system, SEDs due to the DC method can be significantly operated by many mature models and commercial solvents. The ALR approach with relaxing ramp rate relaxation limitations for DED is easily obtained from SED-based solvers. In addition, considering the possible N-1 security limitations and booking needs, the proposed approach can still effectively control this subject as long as it is easier to resolve the relevant SED. In particular, the zero-gap convex relaxation approach for ACOPF is extensively checked in recent years.

2. FORMULATION OF THE DED SUBJECT

DED can be one of the most essential subjects in optimizing the power network operation while searches for the best program for UC. The cost function of the DED subject is to optimize the entire cost of production on the planning interval while is represented as follows [9]:

$$\min \sum_{t=1}^{T} \sum_{j=1}^{N} \alpha_j P_{j,t}^2 + b_j P_{j,t} + c_j \tag{1}$$

where, α_j and b_j are the fuel cost factors of the power plant ith. N is the number of power plants from power production, and $P_{j,t}$ is the output power of the power plant ith in time. T is the entire number of planning times. The hourly power balance is explained using in Equations (1) and (2). The $P_{D,t}$ and $P_{L,t}$ are the entire load and the entire transmission losses of the system over hour t, respectively [9]:

$$\sum_{j=1}^{N} P_{j,t} = P_{D,t} + P_{L,t} \quad \forall t \tag{2}$$

System losses are calculated using B-matrix coefficients in Equation (3). Real power generation limitations are applied using in Equation (4). Ramp-up and ramp-down limitations are applied using in Equations (1)-(5), respectively [9].

$$P_{L,t} = \sum_{j=1}^{N} \sum_{i=1}^{n} P_{j,t} B_{ji} P_{i,t} + \sum_{j=1}^{N} B_{j0} P_{j,t} + B_{00} \quad \forall t \tag{3}$$

$$P_j^{\min} \le P_{j,t} \le P_j^{\max} \quad \forall j, \forall t \tag{4}$$

$$P_{j,t-1} - DR_j \le P_{j,t} \le UR_j + P_{j,t-1} \quad \forall j, \forall t \tag{5}$$

Generation power plants may have a limited operating area based on constraints in the device or instability center. The output product of the generator should be in one of the permitted operating areas. Therefore, this subject formulated as a nonlinear programming (NLP) with disappeared quality [8]:

$$P_{L,t} = \sum_{z=1}^{z_j} P_{(j,t)z} \quad \forall j, \forall t \tag{6}$$

$$P_{(j,t)z} \times P_{j_z}^{\min} \le P_{(j,t)_z}^2 \le P_{(j,t)z} \times P_{j_z}^{\max} \quad \forall j, \forall t, \forall z \tag{7}$$

$$P_{(j,t)i} \times P_{(j,t)n} = 0 \ \forall j, \forall t, \forall = 1,2,...,Z_j - 1, \quad \forall n = i+1,...,Z_j \tag{8}$$

where, Z_j is the number of operating areas allowed. $P_{j_z}^{\min}$ and $P_{j_z}^{\max}$ The high and low constraints of the operating area *Z*th are allowed by power plant *i*. Equations (7) and (8) emphasize only one of the non-zeroing *P* sub-components while zeroing the other sub-components. For utilizing the mentioned problem for POZs, the following integer limitations can be considered to deal with POZs [9]:

$$P_{L,t} = \sum_{z=1}^{Z_j} P_{(j,t)z} \ \forall j, \forall t \tag{9}$$

$$u_{(j,t)z} \times P_{j_z}^{\min} \le P_{(j,t)z} \le u_{(j,t)z} \times P_{j_z}^{\max} \ \forall j, \forall t, \forall z \tag{10}$$

$$\sum_{z=1}^{Z_j} u_{(j,t)z} = 1 \ \forall j, \forall t, \forall z \tag{11}$$

where, $u_{(j,t)z}$ is a binary variable, which related to the *Z*th rated operating area in power plant *i* at hour *T*. This problem becomes a MINLP model.

3. EXECUTION OF OCD (OPTIMIZATION CONDITION ANALYSIS) IN DEAD MODEL

The DED subject is a large-scale NLP or MINLP model in very large practical power networks. Therefore, the execute time (or CPU) is critical to the real-time execution of the DED subject. If the NLP method is used for the DED subject, it can be broken down into some sub-problems with low-dimensional (For example, there are 24 sub-problems for a 24-hour DED.) Therefore, the needed CPU time is significantly decreased. In this chapter, optimization condition decomposition (OCD) is used for this purpose. The OCD method reduces complex limitations in the NLP subject on a large scale. In the case of DED, ramp rate limitations, for example (5), are complex limitations. By reducing these limitations, the reduced subset of *m*th (RSP), (for example, for $t = m$) of DED in the *k*th replication of OCD is as following [9]:

$$\min TC_m^{(k)} = \sum_{j=1}^{N} C_{j,m}\left(P_{j,m}^{(k)}\right) \tag{12}$$

where,

$$C_{j,m}\left(P_{j,m}^{(k)}\right) = a_i C_{j,m}\left(P_{j,m}^{(k)}\right)^2 + b_j P_{j,m}^{(k)} + c_j + \overline{\mu}_{j,m+1}^{UR,(k-1)} + \left(\overline{P}_{j,m+1}^{(k-1)} - P_{j,m}^{(k)} - UR_j\right) + \overline{\mu}_{j,m+1}^{DR,(k-1)} + \left(P_{j,m}^{(k)} - \overline{P}_{j,m+1}^{(k-1)} - DR_j\right) \tag{13}$$

The objective function must be minimized due to the following limitations [9]:

$$\overline{P}_{i,m-1}^{(k-1)} - DR_i \le P_{i,m}^{(k)} \le \overline{P}_{i,m-1}^{(k-1)} + UR_i : \left(\mu_{i,m}^{DR,(k)}, \mu_{i,m}^{UR,(k)}\right) \forall i \tag{14}$$

$$(2),(3),(4),(5),(6),(7),(8) \tag{15}$$

where, $\mu_{i,m}^{DR,(k)}$ and $\mu_{i,m}^{UR,(k)}$ are Lagrangian coefficients that correspond to the complex limitations in Equation (5) of the *m*th sub-problem in repetition *k*th. In Equation (15), $P_{i,t}$ and $P_{L,t}$ with their current equations, i.e. $P_{i,m}^{(k)}$ and $P_{L,m}^{(k)}$ and P_{i,m_z}^{k} are substituted in the corresponding equations. The DED model is solved using the OCD in three levels.

- Step 0. Initial start: In this step, all Lagrangian variables and coefficients are initialized from complex limitations in Equation (5). In this chapter, the primary values of the variables are selected with calculating the DED without considering transmission losses, POZs, and ramp rate limitations. This primary model can be basically a simple *T* convex ED subject.
- Step 1. Solve RSPs: At this stage, the T RSPs are calculated independently and the optimum amounts to entire variables are solved by Lagrangian coefficients of complex limitations in Equation (5).
- Step 2. Stop Criterion: If the variables or the value of the total objective function in Equation (1) do not exchange significantly in two consecutive iterations, the algorithm stops. Otherwise, proceed from step 2. It is worth noting that if the RSPs are solved sequentially rather than in parallel, the recently earned amounts for the variables can be utilized to accelerate the convergence of the method. Namely, if by repeating RSP, k corresponding to interval *m*-1 is calculated before RSP of interval *m*, then it is better to propose the last amounts corresponding to Equation (14). That is, in Equation (14) it should be used $\overline{P}_{i,m-1}^{(k-1)}$ instead of $\overline{P}_{i,m-1}^{(k)}$.

4. LAGRANGE RELEASE METHOD

A mathematical problem by a large number of intervals is in fact a very large Spatio-temporal planning subject that may pose a major computational problem. Dynamic programming, genetic algorithm, simulated annealing approaches, and other heuristic methods are utilized to solve this program. However, the parameter choice significantly affects the performance of these methods. Mathematical programming is robust in engineering subjects [17-20]. To better performance, decomposition methods are proposed to transform the very large model into some small-scale problems. Also, the ALR approach, which is equivalent to a two-step model, has been considered to divide a very large optimization subject with associated nature into several small sub-problems. This method can be convergent if the step size is properly selected to solve the dual problem and several other assumptions are maintained. The Gaussian-Newtonian approach can also be utilized to update coefficients.

To solve the above subjects, the ALR method has been developed, to some extent, robustness and updating, simplicity and stability of coefficient repetition, and in special, for convergence without assumptions, include exact convexity or limitation of the cost function. It can be due to the ALR considers an additional second-class penalty.

Unfortunately, the contributed penalty in the ALR function is non-derivative and leads to the subject of direct analysis compared to the LR approach. Several methods have been proposed to maintain this property. The most common can be the Alternating Direction Method of Multipliers (ADMM), which aims to mix dual ascent breakdown with stronger convergence. In ADMM, the common element of variables is updated periodically or sequentially, where a Gauss-Seidiel pass is used over common variables. Recently, it has proposed the DQAM to approximate the duration of the non-derivation penalty. The optimal solution for convex modeling is also proved. ED storage subjects with complementary limitations are highly non-convex and hard to resolve. In this research, a precise relaxation approach for non-convex ED relaxation in the convex form under two efficient conditions is proposed. Mathematical proof can be proposed and numerical experiments confirm effectiveness of this approach.

The utilize of energy storage systems (ESSs) has been widely considered in the ED of electricity generation. However, complementary limitations, which prevent ESS from simultaneously charging and discharging, must be considered in an ESS ED problem, and this method is highly non-convex and effective in resolving it. Mixed integer programming (MIP) approaches are commonly utilized, with fine penalty approaches, smoothing approaches, and regular relaxation approaches also tested. However, these methods resulted in a high exaction time. Regular relaxation methods are recently considered. Although these approaches do not introduce additional variables and do not result in duplication, they may be challenged under conditions where ESS owners have to pay for power network or ESS restrictions are active. It is, therefore, necessary to consider whether supplementation limitations are completely relaxed for general states. In this study, a precise relaxation approach for convex non-convex ED relaxation under two sufficient conditions has been proposed that can be applied in general cases as well.

4.1. ED Related ESS

horizon extends from $t = 1$ to $t = T$ with an interval of Δt. A storage-related ED model based on the direct current (DC) model can be formulated as follows [10]:

$$\min \sum_{j\in N}\sum_{t\in T}\left(g_j\left(p_j^{dc}(t)\right)-f_j\left(P_j^{ch}(t)\right)\right)+ \sum_{j\in N}\sum_{t\in T}h_i\left(p_i^G(t)\right) \quad (16)$$

For each of the following limitations $t\in T, j\in N, i\in L$ [10]:

$$0\le P_j^{ch}(t)\le \overline{P}_j^{ch}(t),\ \alpha_{j,1(t)},\alpha_{j,2}(t) \quad (17)$$

$$0\le P_j^{dc}(t)\le \overline{P}_j^{dc}(t),\ \alpha_{j,3(t)},\alpha_{j,4}(t) \quad (18)$$

$$E_j(t)=\left(1-\varepsilon_j\right)^t E_j^0+\sum\nolimits_{\tau=1}^{t}\left(1-\varepsilon_j\right)^{t-\tau}\times \left(\eta_j^{ch}P_j^{ch}(\tau)-\frac{P_j^{dc}(\tau)}{\eta_j^{dc}}\right)\Delta t \quad (19)$$

$$E_j^{\min}(t)\le E_j(t)\le E_j^{\max}(t),\ \beta_{j,1}(t),\beta_{j,2}(t) \quad (20)$$

$$\sum\nolimits_{t=1}^{T}\left(1-\varepsilon_j\right)^{T-t}\times\left(\eta_j^{ch}P_j^{ch}(t)-\frac{P_j^{dc}(t)}{\eta_j^{dc}}\right)\Delta t\ge E_j^{\tau},\ \phi_i \quad (21)$$

$$P_j^{ch}(t)P_j^{dc}(t)=0 \quad (22)$$

$$\underline{P}_j^G\le P_j^G(t)\le \overline{P}_j^G \quad (23)$$

$$R_j^{dn}\Delta t\le P_j^G(t+1)-P_j^G(t)\le R_j^{up}\Delta t \quad (24)$$

$$\sum\nolimits_{j\in N}P_j^G(t)+\sum\nolimits_{j\in N}\left(P_j^{dc}(t)-P_j^{ch}(t)\right)= =\sum\nolimits_{j\in N}D_j(t),\quad \lambda(t) \quad (25)$$

$$\underline{P}_i^{Ln}\le\sum_{j\in N}GSF_{i-j}\begin{pmatrix}P_j^G(t)+P_j^{dc}(t)\\ -P_j^{ch}(t)-D_j(t)\end{pmatrix}\le\overline{P}_i^{Ln},\ \mu_{i,1(t)},\mu_{i,2(t)} \quad (26)$$

where, g_j the cost of the ESS discharging is, f_j is the cost of charging the ESS. $f_j\succ 0$ means that the network storage costs to charge, and vice versa if $f_j\succ 0$; h_i is the operating cost of the generator at bus i and $P_j^{dc}(t)$ is the ability to charge and discharge storage on the network side at hour t, $\overline{P}_j^{ch}(t)$ and $\overline{P}_j^{dc}(t)$ are the charge and discharge rate limits, $\eta_j^{ch}(t)$ and η_j^{dc} are charge and discharge efficiencies. $E_j(t)$ is the energy stored in period t, $E_j^{\max}(t)$ and $E_j^{\min}(t)$ are the maximum and minimum constraints of stored energy, E_j^0 is the primary energy ε_j is the rate of self-discharge, E_j^T is the entire charge load, $P_j^G(t)$ is the generation from the generator at hour t, $\overline{P}_j^G$ and $\underline{P}_j^G$ are the maximum and minimum constraints of the output, R_j^{up} and $D_j(t)$ the ramp.are the range of R_j^{dn} is the demand on the bus i at hour t, GSF_{i-j} is the factor of production changes to line j of bus I, $\overline{P}_i^{Ln}$ and are the $\underline{P}_i^{Ln}$ maximum and minimum constraints of transmission capacity. When, $\lambda(t)$, $\alpha_{j,1(t)}$ to $\mu_{i,2(t)}$, $\mu_{i,1(t)}$, $\beta_{j,1}(t),\beta_{j,2}(t)$, $\alpha_{j,4(t)}$ and ϕ_i are the corresponding limitation coefficients.

The model is explanted as follows. The target in (16) is the operation cost of generators and storage devices. Generally, h_i is utilized as a convex second-class function, g_j and f_j are a linear function. So, the problem is a convex model. Assume in Equations (17) and (18) are the range of charge and discharge capacity of storage space. Equation (19) can be an integral relation between stored energy and the previous charge and discharge process from $\tau=1$ to t [10].

Limitation in Equation (20) models the limitation of stored energy, which equals the limitation of charge mode. Limitation in Equation (21) indicates the need for pure charging - especially for collecting electric vehicles (EVs),

E_j^r is the total charge demand. Limitation in Equation (22) is a complementary limitation, which makes the model highly convex. Therefore, the time label is used. Limitations in Equations (23) and (24) represent the production limitations and ramp of a generator. Limitation Equation (25) is the power equality of the power grid over time and Equation (26) represents the limitations of the two-way transmission capacity of the lines.

4.2. Relaxation Conditions

If Equation (22) relaxes, the problem becomes convex. However, the best answer is easily earned. The Karush-Kuhn-Tucker (KKT) conditions are both important and efficient [10]:

$$Con1: \inf g_j'\left(P_j^{dc}(t)\right) \geq \sup f_j'\left(P_j^{ch}(t)\right), \ \forall t$$

$$Con2: f_j'\left(P_j^{ch}(t)\right) < \lambda(t) + \sum_i GSF_j\left(\mu_{i,1}(t) - \mu_{i,2}(t)\right), \ \forall t$$

Proof: Suppose P_j^{ch} and P_j^{dc} exist for storage j at time t in the optimal relaxation method (RM). So $\alpha_{j,1(t)} = 0$ and $\alpha_{j,3(t)} = 0$ due to complementary fertility conditions. Suppose L represents the Lagrangian function of the RM function, ε_j represents $1-\varepsilon_j$ and $\Gamma_t = \sum_{\tau \geq t} \xi_j^{\tau-t}\left(\beta_{j,1}(t) - \beta_{j,2}(t)\right) + \xi_j^{T-t}\varphi_j$. Therefore, using KKT conditions, the following relation is established [10]:

$$\frac{\partial L}{\partial P_j^{ch}(t)} = -f_l'\left(P_j^{ch}(t)\right) - \alpha_{j,1}(t) + \alpha_{j,2}(t) - \eta_j^{ch}\Gamma(t) \Delta t + \lambda(t) + \sum_i GSF_{i-j}\left(\mu_{i,1}(t) - \mu_{i,2}(t)\right) = 0 \quad (27)$$

With $\alpha_{j,1(t)} = 0$, $\alpha_{j,3(t)} \geq 0$ and condition 2, it can be seen that $\varepsilon_t > 0$ is suitable for ESS j at hour t. Utilizing KKT conditions, the following relation is re-established [10]:

$$\frac{\partial L}{\partial P_j^{dc}(t)} = g_l'\left(P_j^{dc}(t)\right) - \alpha_{j,3}(t) + \alpha_{j,4}(t) + \frac{\Gamma(t)\Delta t}{\eta_j^{dc}} - -\lambda(t) - \sum_i GSF_{i-j}(t)\left(\mu_{i,1}(t)\mu_{i,2}(t)\right) = 0 \quad (28)$$

With $\alpha_{j,3(t)} = 0$ by combining Equations (27) and (28), we have [10]:

$$\left(\frac{1}{\eta_j^{dc}} - \eta_j^{ch}\right)\Gamma(t)\Delta t + g_j' - f_j'(t) + \alpha_{j,2}(t) + \alpha_{j,4}(t) = 0 \quad (29)$$

Because of condition 1, it can be inferred that it is also stored for time ESS. So, it can be seen that both $P_j^{ch} > 0$ and $P_j^{dc} > 0$ cannot emerge in the optimal RM method for any ESS at any time interval. However, rest under conditions 1 and 2 is acceptable. The above conditions can be used to the ED model. For example, with renewable energy production considered. This proof also exists for cases not considered in Equation (19).

Specifically, with the cost of charging and discharging as input, the condition 1 can be easily checked. In addition, the condition 1 can be satisfied in two ways. If the ESS belongs to the power system, its operating cost may be ignored during distribution, therefore; Condition 1 is met. Also, if the ESS is owned by a third term, since attract individuals to partnership in the ED, the border compensation paid to them for discharging a power plant of energy must cover the owner's costs of recharging that value of energy. So, $g_j = f_j = 0$, so condition 1 is met. Also, if the ESS is owned with a third term, the border compensation paid to them for discharging a power plant. Therefore, it must continue to satisfy $\inf g_j'\left(P_j^{dc}(t)\right) \leq \sup \sum_j'\left(P_j^{ch}(t)\right)$.

In the case of condition 2, because the marginal value of location on the bus j (LMP_j) in an ED-based DC model can be derived from $\lambda(t) + \sum_i GSF_{i-j}(t)\left(\mu_{i,1}(t) - \mu_{i,2}(t)\right)$ determined.

Condition 2 can be converted $f_j'\left(P_j^{ch}(t)\right) < LMP_j$ meaning that the charge must be (exactly) less than LMP_j at the junction.

If LMP_j or its lower limit can be used using existing historical data, for example, an artificial NN (ANN) method, which is consistent with price predicting in action.

4.3. Parallel LR

A PALR approach is suggested for the DED subject by relaxing the ramp rate limits. Because of the inseparability of the second-class penalty parameter, DQAM can be used to break down the main problem into several sub-problems by similar nature that can been parallel by commercial SED solvers [11].

4.3.1. Formulation of DED Model

The DED subject is comprehensively utilized in the energy management systems and can generally be described as follows [11]:

$$(\text{DED}) \qquad Z = \min_{P_{j,t}} \sum_{j=1}^{N}\sum_{t=1}^{T} f\left(P_{j,t}\right) \qquad \text{(a)}$$

$$\text{s.t.} \quad \sum_{t=1}^{N} P_{j,t} = \sum_{t=1}^{N} D_{j,t} \qquad t = 1,...,T \qquad \text{(b)}$$

$$P_{j,t}^{\min} \leq P_{j,t} \leq P_{j,t}^{\max} \qquad j = 1,...,N \qquad \text{(c)} \quad (30)$$

$$-Rd_{j,t} \leq P_{j,t} - P_{j,t-1} \leq Ru_{j,t}, \ j = 1,...,N \qquad \text{(d)}$$

$$-F_{j,t}^{\max} \leq \sum_{j=1}^{N} S_{l,j}P_{j,t} - \sum_{i=1}^{M} H_{l,j}D_{i,t} \leq F_{l,t}^{\max}, \ l = 1,...,L \quad \text{(e)}$$

That $f(.)$ is a fuel cost function, considered as a second-class function or piece-wise linear function, thus leads to second-class or LP models. Equations (30)b, (30)d and (30)e show the power equality limits, the ramp rate of generator, and the limits of the transmission capacity, respectively. From the DED model, the polygon is surrounded by the T equation, the $NT + L$ inequality, and the elemental limitations NT with the NT variables. This is a high dimension problem, the size of which grows rapidly with increasing time intervals. Also, T is very large, which presents more computational.

Indeed, the cost function in Equation (30)a and limits in Equation (30)c are separable from decision variables $x_1, x_2, \ldots, x_T$, however the linear limitation in Equation (30)d makes them. To dominate the inseparability problem of the bond limitation in Equation (30)d, we consider the ALR method to turn it into a function. While this new model with ALR can be broken down into several isolated small problems and calculated in parallel. This almost reduces computational complexity. Note that the relaxation limitations in the ALR formula are usually used for equality. So, we can enter the fictitious variables $u_{i,t}$ for each limitation in Equation (30)d, as follows [11]:

$$\begin{cases} P_{j,t} - P_{j,t-1} - u_{j,t} = 0 \\ -Rd_{j,t} \le u_{j,t} \le Ru_{j,t} \end{cases} \quad t = 1,...,T, \;\; j = 1,...,N \tag{31}$$

Also, model (1) is written as Equation (32) (a convex model) [11]:

$$\begin{aligned} &\min_{x_1,...,x_T} \; f\left(x_j\right) && \text{(a)} \\ &\text{s.t.} \quad \sum_{j=1}^{T} A_j x_j = b && \text{(b)} \\ &x_j \in \Omega j, \quad j = 1,...,T && \text{(c)} \end{aligned} \tag{32}$$

Cost function in Equation (33) is a convex function [11]:

$$f\left(x_j\right) = \sum_{j=1}^{T} f_j\left(x_j\right) \tag{33}$$

4.3.2. The General Approach to ALR

The augmented Lagrange function can be created as follows [11]:

$$\begin{aligned} L\left(x_j, \pi\right) = {} & f\left(x_j\right) + \pi^T \left(\sum_{j=1}^{T} A_j x_j - b \right) + \\ & + \frac{r}{2} \left(\sum_{j=1}^{T} A_j x_j - b \right)^T \left(\sum_{j=1}^{T} A_j x_j - b \right) \end{aligned} \tag{34}$$

Also, dual function is shown as follows [11]:

$$\begin{aligned} &g\left(\pi\right) = \min_{x_i,\ldots,x_T} L\left(x_j, \pi\right) && \text{(a)} \\ &\text{s.t.} \quad x_j \in \Omega_j, \quad j = 1,\ldots,T && \text{(b)} \end{aligned} \tag{35}$$

while, $A_j \in R^{m\times 1}$ is the Lagrange coefficient due to Equation (32)b, r is a penalty amount and $r > 0$. Similarly, the dual model is represented as follows [11]:

$$\max_{\pi \in R^{m\times l}} g\left(\pi\right) \tag{36}$$

If the problem can be a convex operational problem, there is no problem between the principal subject and the dual, so Equations (36) and (30) are perfectly equivalent. Due to the optimization conditions, as following [11]:

$$\begin{cases} \sum_{j=1}^{T} A_j x_j^* \\ \nabla f_{xj}\left(x_j^*\right) + A_j^T \left(\pi^* + r\left(\sum_{j=1}^{T} A_j x_j^* - b \right) \right) = 0, \; j = 1,\ldots T \end{cases} \tag{37}$$

We can easily obtain the optimal solution:

$$\begin{aligned} &\nabla_{x_i} L\left(x_j, \pi^k\right) = \nabla f_{xj}\left(x_j^{k+1}\right) + A_j^T \\ &\left(\pi^k + \left(\sum_{j=1}^{T} A_j x_j^{k+1} - b \right) \right) = \nabla f_{xj}\left(x_j^{k+1}\right) + A_j^T \pi^{k+1} \end{aligned} \tag{38}$$

So, Lagrange coefficients can be updated with Equations (1)-(39) [11]:

$$\pi^{k+1} = \pi^k + r\left(\sum_{j=1}^{T} A_j x_j^{k+1} - b \right) \tag{39}$$

The stop condition can be defined small enough if the optimal KKT conditions are not possible, such that [11]:

$$\left\| \sum_{j=1}^{T} A_j x_j^{k+1} - b \right\| + \sum_{j=1}^{T} \left\| \nabla f_{xj}\left(x_j^{k+1}\right) + A_j^T \pi^{k+1} \right\| \le \varepsilon \tag{40}$$

where, ε is a definite accuracy.

5. SEMIDEFINITE PROGRAMMINGSOLUTION OF ED

A method involving the formulation of integrated semidefinite programming (SDP) of various ED models is proposed through objective function analysis in this section. The subject of ED is shown as follows [12]:

$$\min C\left(P\right) = \sum_{j=1}^{P} C_j\left(P_j\right) \tag{41}$$

$$\sum_{j=1}^{P} P_j = P_D + P_L\left(P\right), \qquad P = \left[P_1, \ldots, P_P\right]^T \tag{42}$$

$$P_j^{\min} \le P_j \le P_j^{\max} \qquad j = 1,\ldots p \tag{43}$$

The transmission loss is earned utilizing the Kron's loss as following [12]:

$$P_L\left(P\right) = \sum_{j=1}^{P} \sum_{i=1}^{P} P_j B_{ji} P_i + \sum_{j=1}^{P} B_{j01} + P_j + B_{j00} \tag{44}$$

where, B_{ji}, B_{j01} and B_{j00} are the coefficients B. In common ED model management, the objective function is expressed by second-class polynomial functions. Entire fuel cost, $C(P)$ can be explained as [12]:

$$C\left(P\right) = \sum_{j=1}^{P} a_j p_j^2 + b_j P_j + C_j \tag{45}$$

where, P_j is the active product power from the power plant jth, and a_j, b_j and C_j are the corresponding fuel cost factors. Thus, it completes the practical problem considered in this chapter, and presents three properties of thermal power production power plants that apply non-convexity and non-differentiation to the cost function performance.

6. CO-PRODUCTION SCHEME FUEL COST

A contributed cycle power plant includes of one or more gas and steam turbines. The fuel cost is non-smooth and non- derivative [13]:

$$C_j\left(P_j\right)=\begin{cases} b_{j1}P_j+C_{j1}, P_j^{\min}\le P_j\le X_{j1} & \text{(a)}\\ b_{j2}P_j+C_{j2}, X_{j1}\le P_j\le X_{j2} & \text{(b)}\\ b_{jq}P_j+C_{jq}, X_{jq-1}\le P_j\le P_j^{\max} & \text{(c)}\end{cases} \quad (46)$$

while, X_{j1} is the product output of power plant j when that changes from reconfiguration *j* to *j* + 1.

Generation power plants may action on several fuel sources, making it economically viable to use a special fuel for a set of output powers. For this state, the objective function is expressed to show the combination of acceptable fuel options as several convex second-class functions. This complicates determining the most cost-effective type of fuel because the ED model is now discontinuous [14]. The objective function for fuel types *q* can be shown as following [15]:

$$C_j\left(P_j\right)=\begin{cases} b_{j1}P_j^2+C_{j1}, P_j^{\min}\le P_j\le X_{j1} & \text{(a)}\\ b_{j2}P_j^2+C_{j2}, X_{j1}\le P_j\le X_{j2} & \text{(b)}\\ b_{jq}P_j^2+C_{jq}, X_{jq-1}\le P_j\le P_j^{\max} & \text{(c)}\end{cases} \quad (47)$$

The objective function for power plant j is modeled by the point loading of the valve by adding an iterative modified sine component to the second-class objective function [16]:

$$C_j\left(P_j\right)=\left(a_jP_j^2+b_jP_j+C_j\right)+\left|d_j\sin\left(e_j\left(P_j^{\min}-P_j\right)\right)\right| \quad (48)$$

7. MODEL DECOMPOSITION

The separable structure in optimization models breaks down large models to a number of smaller sub-problems that are easier to calculate. Problem analysis is based on objective function or limitations. The superiorities of these analyzes are used to solve ED subjects, but they are focused on the scope of limitation analysis. Although, the observation represents that they are distinguishable between places of non-differentiation in terms of performance. Therefore, the function domain can be subdivided into subdomains under points of non-differentiability. This makes it possible to periodically break down the function into sub-functions whose domains are divided into subdomains, thus eliminating the model of non-derivative. To show this, a non-smooth and non-convex objective function $f\left(\chi\right)$ represented in the operational range $\Omega=[x^{\max},x^{\min}]$, with *n*-1 non-differentiable point, $\chi_1,\chi_2,\ldots,\chi_{n-1}$. The domain $f(\chi)$ can be divided into *n* smaller ranges Ω_i along the points of non-derivative [12].

$$\Omega_j=\left[\chi_{j-1},\chi_j\right],\qquad j=1,\ldots,n \quad (49)$$

where, $\chi_j=\chi^{\min}$ and $\chi_j=\chi^{\max}$, also, suppose that every small range Ω_j is related to a sub-function, $f_j\left(\chi\right)$, for example [12]:

$$f_j\left(\chi\right)=\left\{f\left(\chi\right):x\in\Omega_j\right\} \quad (50)$$

Also, we consider introduction vector $y^T=[y_1,\ldots,y_n]$, where $y_j=\left\{\chi\middle|\chi\in\Omega_j\right\}$. As a result [12]:

$$f\left(\chi\right)=\sum_{j=1}^{n}f_j\left(y_j\right) \quad (51)$$

$$\chi=\sum\nolimits_{j=1}^{n}y_j=1^Ty \quad (52)$$

subject to the complementarity limitation [12],

$$y_j.y_i=0,\ j\ne i \quad (53)$$

while, is equal to the l0-norm cardinality limitation:

$$\|y\|_0=1 \quad (54)$$

It means that a thermal power plant by χ output is divided to smaller power plants, but just one of these smaller power plants can be active at the same time.

Assume a generating power plant *i* with output P_i, the number of fuel options n_i, and a related objective function $C_j\left(P_j\right)$ represented by (80). The generating power plant can be decomposed by the number of outputs $P_{jk},k=1,\ldots,n_j$ to the number of n_i sub-output power plants, whose operating range Ω_{jk} is considered between the points non-differentiability χ_{jk} from $C_j\left(P_j\right)$ [12]:

$$\Omega_{jk}=\left\{p_j/\chi_{jk-1}\le j_i\le\chi_{jk}\right\} \quad (55)$$

Introducing the vector $\bar{P}_j^T=\left[P_{j1},\ldots,P_{jq}\right]$, we write [12]:

$$P_j=1^T\bar{P}_l \quad \text{(a)}$$
$$P_j=\sum\nolimits_{k=1}^{m}P_{jk},\quad P_{jk}\in\Omega_{jk} \quad \text{(b)} \quad (56)$$

By complementarity limitation [12],

$$P_{jr}.P_{js}=0,\qquad r\ne,\qquad r,s=1,\ldots,n_j \quad (57)$$

or equivalent, with a cardinality limitation [12],

$$\left\|\bar{p}_l\right\|_0=1 \quad (58)$$

However [12],

$$C_j\left(P_j\right)\sum_{k=1}^{n_i}C_{jk}\left(P_{jk}\right) \quad (59)$$

where,

$$C_{jk}\left(P_{kj}\right)=a_{jk}P_{jk}^2+b_{jk}P_{jk}+c_{jk} \quad (60)$$

Also, the sub-functions $C_{jk}\left(P_{jk}\right)$ are convex $a_{jk}>0$. A polynomial estimation is considered for modeling the CCCP. The sub-functions are written as following [12]:

$$a_{ik}P_i^2+b_{ik}P_i+c_{ik} \quad (61)$$

Different surrogate functions are utilized in the literature for the modified sine element of Equation (81). These methods are simple but significantly different from the approximate point. A multi-point linearization approach eliminates this shortcoming but increases the size of the problem.

Assume that the sine part of the function corrected by Equation (81) for the ith generating power plant has n_i-1 point of non-derivative, $\chi_{jk},k=1,...,n_j-1$; $P_j^{\min}\le\chi_{jk}\le P_j^{\max}$ earned by solving the problem [12]:

$$\sin\left(e_j\left(P_j^{\min}-P_j\right)\right)=0 \quad (62)$$

to give,

$$\chi_{jk}=P_j^{\min}+\frac{k\pi}{e_j}, \quad k=1,...,n_j-1 \quad (63)$$

By applying the LSE estimation, the sub-functions are proposed for the rectified sine function as follows [12]:

$$\begin{aligned}&\alpha_{j1}P_j^2+\beta_{j2}P_j+\gamma_{j1}, \quad P_j^{\min}\le P_j\le\chi_{j1} && (a)\\ &\alpha_{j2}P_j^2+\beta_{j1}P_i+\gamma_{j2}, \quad \chi_{j1}\le P_j\le\chi_{j2} && (b)\\ &\quad\vdots\quad\vdots\\ &\alpha_{jn_j}P_j^2+\beta_{in_j}P_j+\gamma_{in_j}, \quad \chi_{inj-1}\le P_j\le P_j^{\max} && (c)\end{aligned} \quad (64)$$

Merge the LSE estimation in Equation (64) by the second-class term, we obtain the sub-function for the valve point load objective function as follows [12]:

$$C_j\left(P_j\right)=\begin{cases}a_{j1}P_j^2+b_{j1}P_j+C_{j1}, P_j^{\min}\le P_j\le\chi_{j1} & (a)\\ a_{j2}P_j^2+b_{j2}P_j+C_{j2}, \chi_{j1}\le P_j\le\chi_{j2} & (b)\\ b_{in_j}P_j^2+b_{in_j}P_j+C_{in_j}, \chi_{in_j-1}\le P_j\le P_j^{\max} & (c)\end{cases} \quad (65)$$

That $a_{jk}=a_j+\alpha_{jk}$, $b_{jk}=b_j+\beta_{jk}$, $c_{jk}=c_j+\gamma_{jk}$, $k=1,\dots,n_j$ using χ_{jk} points in (65), the decomposition is in the form Equations (55)-(60). To better estimate, a higher degree polynomial is proposed. For the 4th degree polynomial estimation, the sub-function *k*th of the modified sine function between $\chi_{jk-1}\le P_j\le\chi_{jk}$ is shown as follows [12]:

$$u_{jk}P_j^4+\eta_{jk}P_j^3+\alpha_{jk}P_j^2+\beta_{jk}P_j+\gamma_{jk} \;,\; \chi_{jk-1}\le P_j\le\chi_{jk} \quad (66)$$

And by substituting $W_{jk}=P_{jk}^2$, each of the sub-functions is reduced to a second-class function in W_{jk} and P_{jk} , such as [12]:

$$\begin{aligned}C_{jk}\left(P_{jk},W_{jk}\right)&=u_{jk}W_{jk}^2+\eta_{jk}W_{jk}P_{jk}+\\&+\alpha_{jk}P_{jk}^2+\beta_{jk}P_{jk}+\gamma_{jk},\end{aligned} \quad (67)$$

With second-class limitation [12].

$$W_{jk}=P_{jk}^2 \quad (68)$$

8. SEMIDEFINITE PROGRAMMING (SDP)

In this section, one of the SDP forms is considered as follows [12]:

$$\begin{aligned}&\min \; <A_0,X> && (a)\\ &\text{subject to } <A_j,X>b_j \quad j=1,\dots,m && (b)\\ &X\ge 0 && (c)\end{aligned} \quad (69)$$

The SDP dual problem is as follows [12]:

$$\begin{aligned}&\max \; <b,y> && (a)\\ &\text{subject to } \sum_{j=1}^{m}y_jA_j\le A_0, y\in R^m && (b)\end{aligned} \quad (70)$$

while, $X\in S^n$ is the decision vector, $b\in R^n$ and $A_0,A_i\in S^n$. The S^n can be the set of entire symmetric matrices in $R^{n\times n}$.

By introducing a variable of the semidefinite matrix in direct formulation as SDP in some subjects, it does not always have a convex SDP model. The causes given for it consist the possible non-convexity in the limitation set (for example rank limitation, disjoint limitation set) or in the cost function. Therefore, the obtained SDP model is hard to apply in the best optimal solution. The SDP subject can be relaxed by embedding a non-convex limitation in a larger convex limitation set. Assume the following relation.

$$f^*=\min_{x\in\Omega'}g\left(x\right) \quad (71)$$

The problem becomes relaxed [12]:

$$f_*=\min_{x\in\Omega'}g\left(x\right) \quad (72)$$

Since the ED model is quadratically constrained second-class model (QCQP), its LR is proposed to represent the topic. Assume the QCQP model [12]:

$$\begin{aligned}&\min \; f_0\left(x\right) && (a)\\ &\text{subject to } f_j\left(x\right)\ge 0 \quad j=1,\dots,p && (b)\end{aligned} \quad (73)$$

$$\begin{aligned}f_j\left(x\right)&=x^TA_jx+2b_j^Tx+c_j && (a)\\ &=\bar{x}^TM_1\bar{x} && (b)\\ &=Tr\left(M_j\bar{x}\bar{x}^T\right) && (c)\end{aligned} \quad (74)$$

$$M_j=\begin{bmatrix}A_j & b_j\\ b_j^T & C_j\end{bmatrix}, \quad \bar{x}=\begin{bmatrix}x\\1\end{bmatrix} \quad (75)$$

The model takes SDP description as following [12]:

$$\begin{aligned}&\min \; <M_0,X> && (a)\\ &\text{subject to } <M_i,X>\ge 0, \quad i=1,\dots,p && (b)\\ &X>0 && (c)\end{aligned} \quad (76)$$

$$X=\bar{x}\bar{x}^T; \quad rank\left(X\right)=1 \quad (77)$$

The QCQP subject Equation (73) can be equivalent for the SDP model n Equation (76) with the rank limit, rank (*X*) = 1. Limits the rank of non-convexity to the SDP model.

Removing the rank limitation in the SDP model guarantees convexity, however the result for relaxing subject is no longer equal to the main model. Because the application of rank limitations is essential for equal subject, it is necessary to correct the rank limitation. A convex reformulation that imposes a limitation, called convex iteration. This formulization decreases the non-convex rank-constrained SDP model [12].

$$\begin{aligned}&\min \; <A_0,X> && (a)\\ &\text{subject to } <A_j,X>=b_j, \quad j=1,\dots,q && (b)\\ &X\ge 0, \quad rank\left(X\right)=1 && (c)\end{aligned} \quad (78)$$

For a convex subject [12]:

$$\begin{aligned}&\min \; A_0.X+w<W^*,X> && (a)\\ &\text{subject to } <A_j,X\ge b_j, \quad j=1,\dots,q && (b)\\ &X\ge 0 && (c)\end{aligned} \quad (79)$$

where, the matrix with direction W^* is the best answer for a convex model [12]:

$$\begin{aligned} &\min \ <W, X^*> && (a) \\ &\text{subject to } <I, W>= q-1, \quad j=1,\dots,q && (b) \\ &I \geq W \geq 0 && (c) \end{aligned} \tag{80}$$

where, X^* is the last answer of the convex model in Equation (79). Repeat the method and solve two convex models alternately when the stop scale $\langle X^*, W^* \rangle = 0$ is obtained. The optimization model in Equation (79) starts with $W^* = I$. The like approach can be the nuclear norm or trace heuristic method. It is also represented that the trace heuristic method can be the first iteration of the convex iteration method. So, when there is no solution for rank 1, we have both methods to provide a resource.

Branches and boundaries for SDP are considered to be bound by the combined result of relaxation approaches, i.e., the decomposition of the non-convex term and the replacement of the convex iteration for the rank limitation obtains the convex ED model of a piece for global optimization. However, this does not apply to cases with piecewise non-convex functions. Therefore, an alternative path is needed to obtain optimal solutions such as non-convex models. This method solves the SDP model for the optimal solution. According to the breakdown of all objective functions, the ED model in Equation (41) can be written as follows [12]:

$$\begin{aligned} &\min \sum_{j=1}^{m}\sum_{i=1}^{n_i}\left[a_{ji}P_{ji}^2 + b_{ji}P_{ji} + c_{ji}\right] && (a) \\ &\text{s.t. } \sum_{j=1}^{m}\sum_{j=1}^{n_i} P_y = P_D && (b) \\ &P_{jr}.P_{js} = 0, \quad r \neq s; \quad j=1,\dots m; r,s=1,\dots n_{j;} && (c) \\ &\sum_{i=1}^{n_j} P_{ji} \geq P_{j1}^{\min}, \quad j=1,\dots,m && (d) \\ &P_{ji}^{\min} \leq P_{ji} \geq P_{ji}^{\max}, \quad j=1,\dots,m, \quad i=1,\dots,n_i && (e) \end{aligned} \tag{81}$$

$$\begin{aligned} &\min \ \sum_{j=1}^{m} x_j^T A_{0j} x_j + 2b_{0j}^T x_j + c_{0j} && (a) \\ &\text{s.t. } \sum_{j=1}^{m} 1^T x_j = P_D && (b) \\ &\left\|x_j\right\|_0 \leq 1; \quad j=1,\dots,m; && (c) \\ &1^T x_j \geq P_j^{\min}; \quad j=1,\dots,m && (d) \\ &\chi_j^l \leq x_j \geq \chi_j^u; \quad j=1,\dots,m && (e) \end{aligned} \tag{82}$$

The positive semi-definite symmetric matrices X_j for each power plant i is as follows [12]:

$$X_j = \begin{bmatrix} x_j \\ 1 \end{bmatrix}\begin{bmatrix} x_j \\ 1 \end{bmatrix}^T = \begin{bmatrix} x_j x_j^T & x_j \\ x_j^T & 1 \end{bmatrix}; \tag{83}$$
$$rank\left(X_j\right) = 1,$$

Therefore, the SDP formula of the problem in Equation (81) can be written as follows [12]:

$$\begin{aligned} &\min \sum_{j=1}^{m} <\bar{A}_{0j}, X_j> && (a) \\ &\text{s.t. } \sum_{j=1}^{m} <\bar{A}_{ji}, X_j> \geq 0, \quad j=1,\dots,n_c && (b) \\ &X_j > \geq 0, \quad X_j\left(n_j+1, n_j, 1\right) = 1, \quad j=1,\dots,m && (c) \\ &rank\left(X_j\right) = 1, \quad \left\|x_j\right\|_0 = 1, \quad j=1,\dots,m && (d) \end{aligned} \tag{84}$$

The A_{ji} matrix is constructed as follows [12]:

$$\bar{A}_{ji} = \begin{bmatrix} A_{ji} & b_{ji} \\ b_{ji}^T & C_{ji} \end{bmatrix}, \quad j=0,1,\dots,m; \quad i=0,1,\dots,n_c \tag{85}$$

By using convex iteration, we will have [12]:

$$\begin{aligned} &\min \sum_{j=1}^{m}\left(\bar{A}_{0j}, X_j > + \omega \sum_{j=1}^{m} < G_j W_j^* >\right. && (a) \\ &\text{subject to } \sum_{j=1}^{m} < \bar{A}_{ij}, X_j \geq 0, \quad i=1,\dots,n_c && (b) \\ &X_i \geq 0 && (c) \end{aligned} \tag{86}$$

$$\begin{aligned} &\min \ \sum_{j=1}^{n} < \bar{A}_{0j}, X_j^* > + \omega < G_j^*, W_j > && (a) \\ &\text{subject to } < I, W_j \geq n_j - 1 && (b) \\ &I \geq W_j \geq 0 && (c) \end{aligned} \tag{87}$$

➢ Example: In Table 1 and Figure 1 [23], the information of power plants and hourly load demand are shown, respectively. Solve the DED problem for these generators first using the MINLP method presented in section 1.2. Then solve this problem using the convex optimization method presented in section 1.8.3 and compare the results.

Table 1. The data for four power plants [23]

g_i	a_i	b_i	c_i	d_i	e_i	f_i (kg)	$P_{g,i}^{\min}$ (MW)	$P_{g,i}^{\max}$ (MW)	RU_i (MW)	RD_i (MW)
1	0.12	14.8	89	1.2	-5	3	28	200	40	40
2	0.17	16.57	83	2.3	-4.24	6.09	20	290	30	30
3	0.15	15.55	100	1.1	-2.15	5.69	30	190	30	30
4	0.19	16.21	70	1.1	-3.99	6.2	20	260	50	50

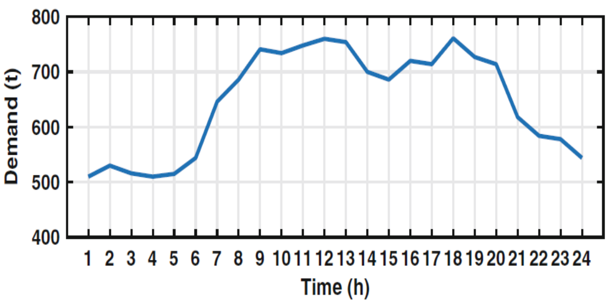

Figure 1. The hourly demand [23]

Figures 2 and 3 show the results for MINLP and convex optimization methods, respectively. Comparing these results, we see that in the MINLP method, the power of power plant 3 is different from 20h to 24 h compared to the convex optimization method. However, in other hours and at other power plants, there is little difference between the tw1o methods. The cost function value for the MINLP method is 648019.0591$. This is 647964.4601$ for a convex optimization method. Therefore, the convex optimization method has a better performance than the nonlinear model, which represents the performance of the convex model to calculate the DED model.

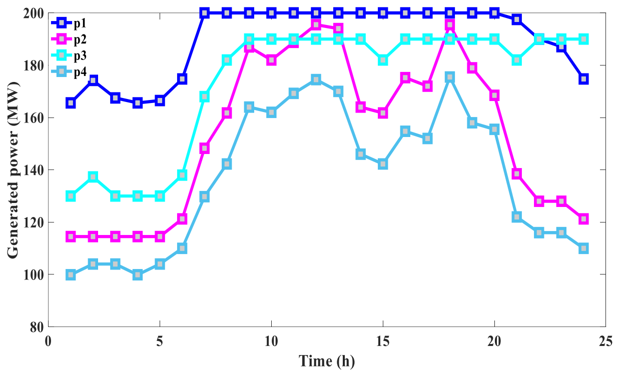

Figure 2. The hourly thermal power plant power schedules with MINLP method

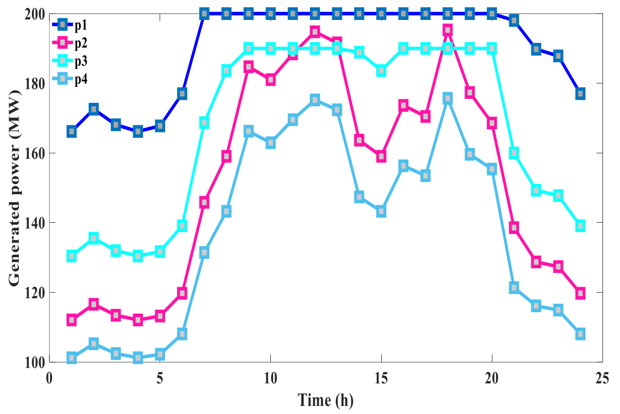

Figure 3. The hourly thermal power plant power schedules with convex optimization method

9. CONCLUSION

In this chapter, the solution of the ED model using Lagrange relaxation methods and convex optimization is investigated. The Lagrange relaxation method is divided into several small sub-problems to divide an optimization model. To obtain the problems of the Lagrange relaxation method, the ALR method has been strengthened, developed for precise convergence and convexity. This is because the ALR considers an additional second-class penalty, the benefit of which is that the dual model can be detected on the main problem in relatively mild conditions. Also, the DQAM method is proposed for the convex optimization of the ED subject. Global convergence for convex modeling has also been proven and successfully utilized in calculations. Also, two MINLP methods and a convex optimization method are implemented in Gam's software. The results represent the better performance of the convex optimization method in solving the DED problem.

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BIOGRAPHIES

Mahdi Hedayati was born in Mashhad, Iran, 1969. He received the B.Sc. from Ferdowsi University, M.Sc. from Islamic Azad University and Ph.D. from UPM in 1991, 1996 and 2012 respectively all in the field of Electrical Power Engineering. Currently, he is Assistant Professor at Electrical Engineering Department of Karaj Branch, Islamic Azad University. His research interests are Power Electronics, Modelling and Control of Power Systems.

Reza Effatnejad was born in Abadan, Iran, 1969. He received the B.Sc. from K.N. Toosi, M.Sc. degrees from Amirkabir University and the Ph.D. degree from Iran University of Science and Technology in Electrical Engineering. He is an Assistant Professor of Power Electrical Engineering at Islamic Azad University – Karaj Branch. He has more than 50 paper in Energy Efficiency, Renewable Energy and Power Engineering.

Keyvan Choopani was born in Rasht, Iran on April 8, 1990. He received the B.Sc. degree in electrical engineering from Ekbatan University, Qazvin, Iran, in 2012, the M.S. degree in electrical engineering from Islamic Azad University Science and Research Branch, Alborz, Iran, in 2015, and the Ph.D. degree from the Islamic Azad University, Karaj Branch, Alborz, Iran, in 2020. He has many paper in Smart Grid, Renewable Energy and Micro-Grid. His research interests are Micro Grids, Smart Grids, Self-healing, Demand Response and Operation of Power Systems. In addition to teaching at the university, he is in the distribution system design department of Tehran province.

Fatemeh Koneshloo was born in Tehran, Iran on July 24, 1990. She received the B.Sc. degree in electrical engineering from Shahab Danesh University, Qom, Iran, in 2016, the M.S. degree in electrical engineering from University of Ghiaseddin Jamshid Kashani, Qazvin, Iran, in 2018. Her research interests are Micro Grids and Smart Grids.

Mohammad Effatnejad Was born in Tehran. In 1996. He received B.Sc. in Electrical Engineering Department, Civil Aviation Technology College, 2021. Now, He is M.Sc. student in Faculty of Management and Economic (MBA), Science and Research Branch, Islamic Azad University. His research interest is in communication, Electrical System, Modeling, Parameter Estimation, Management and Economic. He published papers in international journal and IEEE conferences and book chapter in Springer.

Ali Effatnejad was born in Tehran, Iran, 2001.He is student in Faculty of Medical, Shahrood Branch, Islamic Azad University. His research interest is Medical and Medical Engineering, Intelligent system. and documentary of books and papers. He has paper in Medical Science.