International Journal on
"Technical and Physical Problems of Engineering"
(IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

June 2023 | Issue 55 | Volume 15 | Number 2 | Pages 175-180

REAL ECONOMIC DISPATCHING USING FLETCHER-REEVES METHOD

A. Bouadi [1]   A. Mostefa [2]   M. Rahli [2]

1. *Department of Physic, University of Sciences and Technology of Oran, Oran, Algeria, abed.bouadi@univ-usto.dz*
2. *Electrical and Automatic Department, University of Relizane, Relizane, Algeria*
abdelkader.mostefa@univ-relizane.dz, mostefa.rahli@univ-relizane.dz

Abstract- In this paper, we will apply a method of non-linear programming to the economic dispatching of the active powers in an electric network. This method was described by Fletcher-Reeves [1, 2] and from the application point of view to the study of the electrical supply network. We used it for the first time to solve the real economic dispatching. This method derives from the method of the combined gradient and by using the development of Taylor. We end to a final form where we can determine the optimum capacities to make lower the production cost of the energy requested by non-linear programming (PNL).

Keywords: Economic Dispatch, Nonlinear Programming, Power System, Penalty Function.

1. INTRODUCTION

The required production of electrical energy becomes more and more important. It will be necessary to reflect on a strategy of production of this energy for lower cost. For that, several researchers succeeded in developing mathematical methods which make the objective function minimum. In our case we used a method of programming non-linear which is the method of Fletcher-Reeves [1; 2] that we applied for the first time to standard network A.E.P 14 bus [4] and compared the results found with those already found by others methods [5].

2. OPTIMAL DISTRIBUTION BY THE NON-LINEAR PROGRAMMING

The function objectifies which is the fuel performance index is of origin non-linear and its form is [1, 3, 4]:

$$F_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{1}$$

The principal task to solve is summarized as:

$$\text{Min}\left\{F_i(P_{Gi})\right\} \tag{2}$$

with the constraints:

$$P(P_{Gi}, r^{(k)}) = F(P_{Gi}) + \frac{1}{r^{(k)}}\left(\sum_{i=1}^{n} H(h_i(P_{Gi}))\right) + r^{(k)}\left(\sum_{i=1}^{m} G(g_i(P_{Gi}))\right) \tag{3}$$

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \tag{4}$$

where, a_i, b_i, c_i: performance index of production of power station i

C_j: consumption of power activates in the node j

P_L: total real power losses

$P_{Gi}^{\min}$: Minimal active power of power bus i

$P_{Gi}^{\max}$: Maximum active power of power bus i

For solving this task, we used a new method of non-linear programming as we already underlined and who is the method of Fletcher-Reeves.

3. MATHEMATICAL FORMULATION OF THE PROBLEM

The problem of the economic control system consists in minimizing the function of the production cost of electric power (objective function) and this problem perhaps stated as:

To minimize the $F(P_{Gi})$ function (5)

Under the constraints:

$$h_i(P_{Gi}) = 0 \quad (i=1,2........., n) \tag{6}$$

$$g_i(P_{Gi}) \ge 0 \quad (i=1,2........., m) \tag{7}$$

As our task proves to be an optimization with constraints, it should be transformed into another similar but without countering by using the functions of penalty. The initial problem changes by another which are:

$$\text{Minimize } P(P_{Gi}, r^{(k)}) = F(P_{Gi}) + \frac{1}{r^{(k)}}\left(\sum_{i=1}^{n} H(h_i(P_{Gi}))\right) + r^{(k)}\left(\sum_{i=1}^{m} G(g_i(P_{Gi}))\right) \tag{8}$$

where, $r^{(k)}$ is the coefficient of penalization.

The functions H (h_i (P_{Gi})) and G (g_i (P_{Gi})) are called functions of penalization and are defined according to the method of penalty used.

3.1. Methods of External Penalties

$G(g_i(P_{Gi})) = \left[G_i(P_{Gi})\right]^2$ and $H(h_i(P_{Gi})) = \left[H_i(P_{Gi})\right]^2$

Therefore:

$$P(P_{Gi}, r^{(k)}) = F(P_{Gi}) + \frac{1}{r^{(k)}}\sum_{i=1}^{n}\left[h_i(P_{Gi})\right]^2 + \left(r^{(k)}\right)\sum_{i=1}^{m}\left[g_i(P_{Gi})\right]^2 \tag{9}$$

3.2. Methods of Interior Penalties

These methods are based only on the transformation of the problem with constraints of inequality type [1, 2].

$$G\left[g_i(P_{Gi})\right] = \frac{1}{g_i(P_{Gi})} \quad (10)$$

Therefore:

$$P(P_{Gi}, r^{(k)}) = F(P_{Gi}) + r^{(k)} \sum_{i=1}^{m} \left(\frac{1}{(g_i(P_{Gi}))} \right) \quad (11)$$

3.3. Methods of Penalties Mixed [1, 2]

It is a combination of the two preceding methods:

$$H(h_i(P_{Gi})) = \left[H_i(P_{Gi})\right]^2 \quad \text{and} \quad G\left[g_i(P_{Gi})\right] = \frac{1}{g_i(P_{Gi})}$$

Therefore:

$$P(P_{Gi}, r^{(k)}) = F(P_{Gi}) + \frac{1}{r^{(k)}} \sum_{i=1}^{n} \left[h_i(P_{Gi})\right]^2 + \left(r^{(k)}\right) \sum_{i=1}^{m} \left(\frac{1}{g_i(P_{Gi})} \right) \quad (12)$$

4. METHOD OF THE COMBINED GRADIANT

4.1. General Principle

They are iterative methods applied to a non-linear function and while benefiting from the development of this function in Taylor series in the vicinity of $P_{Gi}^{(k)}$ and by neglecting the 3ieme term as well as the terms of a higher nature, so the new form of the function to minimize becomes [2];

$$F(P_{Gi}) = F(P_{Gi}^{(k)} + \nabla^T F(P_{Gi}^{(k)})(P_{Gi} - P_{Gi}^{(k)}) + \frac{1}{2}(P_{Gi} - P_{Gi}^{(k)})^T \nabla^2 F(P_{Gi}^{(k)}(P_{Gi} - P_{Gi}^{(k)}) \quad (13)$$

at the stage (k+1) 2nd iteration we can affirm that:

$$F(P_{Gi}^{(k+1)}) = F(P_{Gi}^{(k)} + \nabla^T F(P_{Gi}^{(k)})(P_{Gi}^{(k+1)} - P_{Gi}^{(k)} + \frac{1}{2}(P_{Gi}^{(k+1)} - P_{Gi}^{(k)T} \nabla^2 F(P_{Gi}^{(k)}(P_{Gi}^{(k+1)} - P_{Gi}^{(k)}) \quad (14)$$

$P_{Gi}^{(k+1)}$ is a minimum if $\nabla F(P_{Gi}^{(k+1)}) = 0$

This condition leads to the following linear system:

$$\nabla F(P_{Gi}) = -\nabla^2 F(P_{Gi}^{(k)}(P_{Gi}^{(k+1)} - P_{Gi}^{(k)}) \quad (15)$$

From where the iterative formula is:

$$P_{Gi}^{(k+1)} = P_{Gi}^{(k)} - \left[\nabla^2 F(P_{Gi}^{(k)}\right]^{-1} \nabla F(P_{Gi}) \quad (16)$$

$$P_{Gi}^{(k+1)} = P_{Gi}^{(k)} - H^{-1} \nabla F(P_{Gi}) \quad (17)$$

$$P_{Gi}^{(k+1)} = P_{Gi}^{(k)} + S^k \quad (18)$$

where,

$$S^{(k)} = -H^{-1} \nabla F(P_{Gi}) \text{ (Direction of research)} \quad (19)$$

$$H = \nabla^2 F(P_{Gi}^{(k)}) \text{ (Hessian matrix)} \quad (20)$$

This formula perhaps generalized by introducing a factor $\lambda^{(k)}$ acceleration of convergence into the direction of research:

$$P_{Gi}^{(k+1)} = P_{Gi}^{(k)} + \lambda^{(k)} S^k \quad (21)$$

$$\lambda^{(k)} = -\nabla^T F(P_{Gi}^{(k)}) \frac{S^k}{S^{kT}} \nabla^2 F(P_{Gi}^{(k)}) + S^k \quad (22)$$

4.2. Method of Fletcher-Reeves

The method of Fletcher-Reeves is a direct extension of the method of the combined gradient where a scalar weight $\omega_{(k)}$ being selected so that the direction of research $S^{(k)}$ is combined compared to all the preceding directions and the new direction is calculated starting from the new gradient [1, 2, 5, 6, 8, 11].

An initial direction is taken: $S^{(0)} = -\nabla F(P_{Gi}^{(0)})$ and $P_{Gi}^{(1)} - P_{Gi}^{(0)} = -\lambda^{(0)}$ then to make:

$$S^{(1)} = -\nabla F(P_{Gi}^{(1)}) + \omega^{(1)} S^{(0)} \quad (23)$$

where,

$$\omega^{(1)} = \nabla^T F(P_{Gi}^{(1)}) \frac{\nabla F(P_{Gi}^{(1)})}{\nabla^T F(P_{Gi}^{(0)})} \nabla F(P_{Gi}^{(0)}) \quad (24)$$

From where the iterative formula is:

$$S^{(k+1)} = -\nabla F(P_{Gi}^{(k+1)}) + S^{(k)} \nabla^T F(P_{Gi}^{(k+1)}) \frac{\nabla F(P_{Gi}^{(k+1)})}{\nabla^T F(P_{Gi}^{(k)})} \nabla F(P_{Gi}^{(k)}) \quad (25)$$

4.3. Algorithm of Resolution

a) Stage 0: $P_{Gi}^{(0)}$ is the starting point chosen, to pose $S^{(0)} = -\nabla F(P_{Gi}^{(0)})$

b) Stage K: to choose $\lambda^{(k)}$

minimizing: $F(\lambda^{(k)}) = F(P_{Gi}^{(k)}) + \lambda^{(k)} S^{(k)}$ to pose:

$$P_{Gi}^{(k+1)} = P_{Gi}^{(k)} + \lambda^{(k)} S^{(k)}$$

$$S^{(k+1)} = -\nabla F(P_{Gi}^{(k+1)}) + S^{(k)} \nabla^T F(P_{Gi}^{(k+1)}) \omega^{(k+1)}$$

with: $\omega^{(k+1)} = \dfrac{\nabla^T F(P_{Gi}^{(k+1)}) \nabla F(P_{Gi}^{(k+1)})}{\nabla^T F(P_{Gi}^{(k)}) \nabla F(P_{Gi}^{(k)})}$

c) Test of Stop: if checked: End.

so not, to make $k = k$+1 and to turn over in (b).

5. EXPERIMENTATION

For application of this method, we used standard network A.E.P 14 Bus Test System [4, 9, 10].

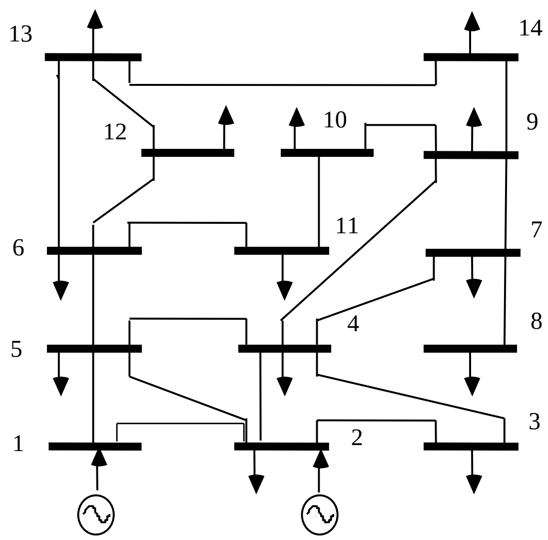

Figure 1. AEP 14 units test system

The performance indexes of the two nodes of production are:

$$F_2(P_{G2}) = 130 + 2.1P_{G2} + 0.009P_{G2}^2$$

$$F_1(P_{G1}) = 100 + 1.5P_{G1} + 0.006P_{G1}^2$$

With the following constraints:

$135 \le P_{G1} \le 195$ (MW)

$70 \le P_{G2} \le 145$ (MW)

$\sum C_k = 259$ (MW)

Initially, we supposed the losses which were calculated by the method of Gauss-Seidel as being constant and do not depend on any variable. In the second place which is the second alternative, we supposed this time, the power losses are a linear function of the generated powers. In each alternative, we used the two forms of external and mixed penalty to see their influence on the found results. This fact we compared the results of each alternative.

The found results are illustrated by the Tables 2 and 3 for the first alternative and the Tables 4 and 5 for the second alternative.

5.1 Calculation with the Constant Losses (First Alternative)

It is considered that the active losses are constant; the calculation of the flow of power by the method of Gauss-Seidel gave like total losses of the network gives [1, 3]:

P_L = 18.85 (MW)

The constraint of the equality type becomes:

$\sum P_{Gi}$ = 277.85 (MW)

We fix the initial values P_{G1}^0 and P_{G2}^0 carries out the calculation of optimum capacities.

We vary the initial values progressively (fifteen times) in order to observe their influence on the optimal solutions. The found results are in the Tables 2 and 3 and illustrated by Figures 2 and 3.

Table 2. Method of external penalty

P_{G1}^0 MW	P_{G2}^0 MW	P_{G1}^{opt} MW	P_{G2}^{opt} MW	$Cost^{opt}$ $/h	Time s
140	105	163.79	112.15	985.39	0
140	110	162.54	113.55	986.39	0
140	115	161.42	114.71	987.77	0
145	105	164.75	111.44	985.79	0
145	110	163.42	112.77	986.72	0
145	115	162.64	113.76	988.05	0
150	105	165.69	110.75	986.22	0
150	110	164.45	112.01	987.08	0
150	115	164.08	112.61	988.88	0
155	105	166.60	110.10	986.72	0
155	110	165.53	111.18	987.45	0
155	115	166.36	110.64	988.14	0
160	105	167.38	109.58	987.37	0
160	110	166.85	110.14	987.78	0
160	115	166.59	110.65	988.98	0

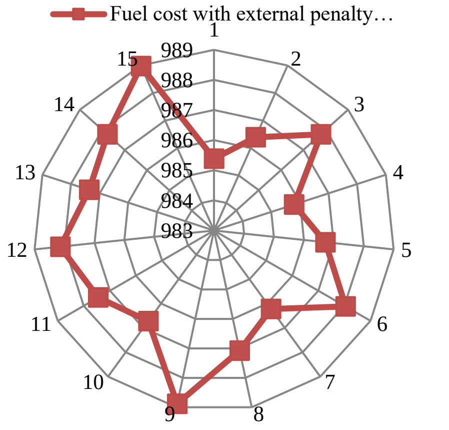

Figure 2. Fuel cost with external penalty in first case

Table 3. Method of penalty mixed

P_{G1}^0 MW	P_{G2}^0 MW	P_{G1}^{opt} MW	P_{G2}^{opt} MW	$Cost^{opt}$ ($/h)	Time (s)
140	105	156.367	121.05	997.34	0
140	110	153.905	123.53	999.73	0
140	115	151.464	125.10	1002.31	0
145	105	158.855	118.56	995.19	0
145	110	156.400	121.04	997.40	0
145	115	153.977	123.49	999.82	0
150	105	161.348	116.07	993.23	0
150	110	158.905	118.54	995.26	0
150	115	156.525	120.97	997.51	0
155	105	163.846	113.58	991.45	0
155	110	161.428	116.03	993.32	0
155	115	159.193	118.35	995.42	0
160	105	166.352	111.08	989.87	0
160	110	164.017	113.47	991.60	0
160	115	163.343	114.37	993.01	0

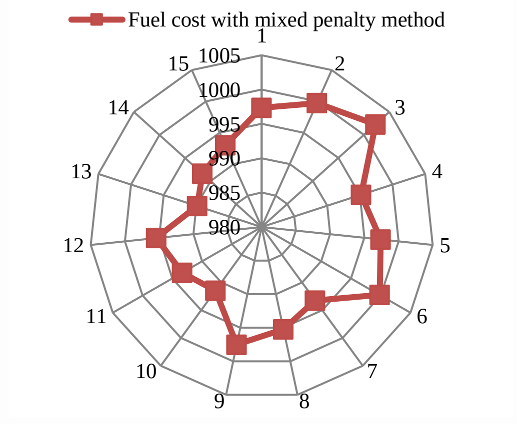

Figure 3. Fuel cost with mixed penalty in first case

5.2. Calculation with the Variable Active Losses (2rd Alternative)

In this case the power losses are regarded as being a linear function of the generated powers.

The coefficients of the generated powers are determined by the method of Gauss-Seidel [1].

$P_L = 0.07677\,P_{G1} + 0.01476\,P_{G2}$ (MW)

The equation of assessment will thus become:

$0.92323\,P_{G1} + 0.98524\,P_{G2} = 259$ (MW)

Table 4. Method of external penalty

P_{G1}^{0} MW	P_{G2}^{0} MW	P_{G1}^{opt} MW	P_{G2}^{opt} MW	$Cost^{opt}$ \$/h	P_L MW	*Time* s
140	105	160.04	112.71	974.79	13.95	0
140	110	159.39	113.67	976.54	13.91	0
140	115	161.06	112.94	979.26	14.03	0
145	105	161.20	111.54	973.95	14.02	0
145	110	160.59	112.47	975.66	13.98	0
145	115	164.89	108.55	974.49	14.26	0
150	105	162.35	110.38	973.17	14.09	0
150	110	161.82	111.23	974.84	14.04	0
150	115	161.85	109.68	968.59	14.04	0
155	105	163.51	109.23	972.45	14.16	0
155	110	163.22	109.87	974.08	14.15	0
155	115	164.35	108.26	971.44	14.21	0
160	105	164.65	108.10	971.84	14.23	0
160	110	165.31	107.59	972.10	14.27	0
160	115	162.63	110.21	973.42	14.11	0

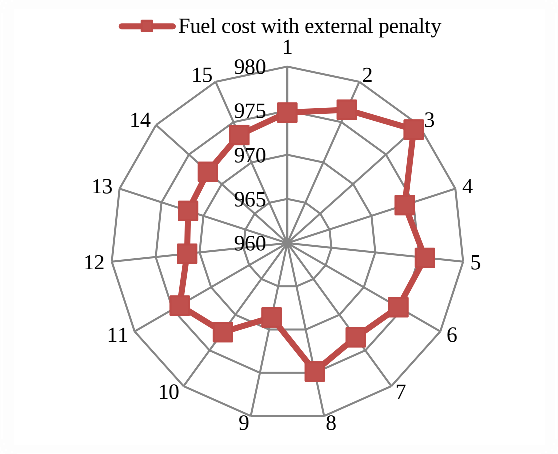

Figure 4. Fuel cost external penalty in second case

The results of the calculation of the optimal generated active powers, of the minimal fuel cost, the total losses and the computing time are given by the Tables 4 and 5 and illustrated by Figures 4-7.

Table 5. Method of penalty mixed

P_{G1}^{0} MW	P_{G2}^{0} MW	P_{G1}^{opt} MW	P_{G2}^{opt} MW	$Cost^{opt}$ \$/h	P_L MW	*Time* s
140	105	153.6	118.4	976.76	13.54	0
140	110	151.1	120.8	978.49	13.38	0
140	115	148.5	123.2	980.40	13.22	0
145	105	156.1	116.0	975.31	13.70	0
145	110	153.6	118.4	976.85	13.54	0
145	115	151.1	120.8	978.58	13.38	0
150	105	158.8	113.6	974.03	13.86	0
150	110	156.2	116.0	975.40	13.70	0
150	115	153.7	118.4	976.94	13.54	0
155	105	161.4	111.2	972.94	14.02	0
155	110	158.8	113.5	974.12	13.86	0
155	115	156.5	115.8	975.43	13.72	0
160	105	163.9	108.8	972.03	14.19	0
160	110	161.5	111.1	973.02	14.03	0
160	115	158.8	113.6	974.15	13.86	0

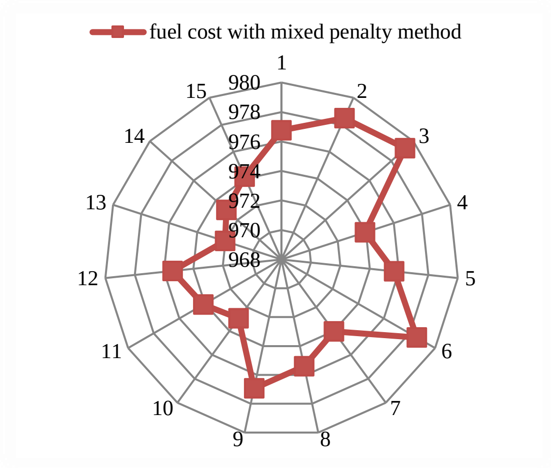

Figure 5. Fuel cost mixed penalty in second case

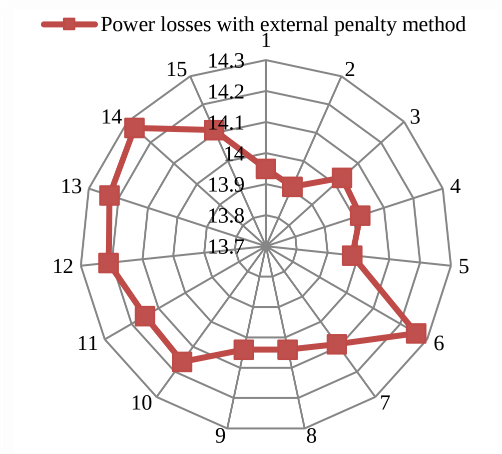

Figure 6. Power losses external penalty in second case

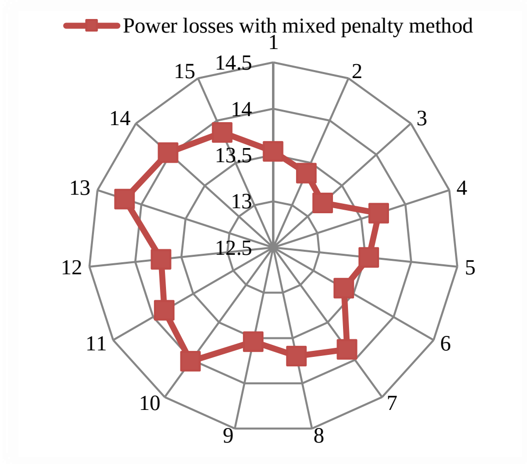

Figure 7. Power losses mixed penalty in second case

6. CONCLUSION

We can conclude starting from the results found on the level from the two alternatives with the two types of penalty that:

– The optimal generated powers as well as the production cost are insensitive with the initial values i.e., for all the interval of variation there is a variation moreover at least 5 (MW).

– The optimal cost of production is better for the second alternative.
– The losses are minimal when they are expressed according to transform the initial problem to the second form which is function of the generated powers.
– The computing time is practically zero.
– Among the other studied methods, we noted that the method of Fletcher Reeves gives a minimum cost compared to the others like its speed of convergence for the same initial approximation.
– We also noticed that the losses for the new method compared to the others are found minimal by the other methods, so, the second alternative is better than the first.

REFERENCES

[1] M. Rahli, "Economic Dispatching by Linear and Nonlinear Programming", Ph.D. Thesis, U.S.T.O, pp. 39-49, Oran, Algeria, January 1996.
[2] D.M. Himmelblau, "Applied Non-linear Programming", Mc Graw-Hill Book Company, pp. 303-319, New York, USA, 1972.
[3] G.W. Stagg, A.H. El Abiadh, "Methods Computer in Power System Analysis", Mc Graw-Hill Book Company, pp. 261-266, New York, USA, 1968.
[4] L.L Freris, A.M. Sasson, "Investigation of the Load Flow Problem", I.E.E., Flight 115, No. 10, pp. 1459-1470, October 1968.
[5] L. Abdelmalek, M. Zerikat, M. Rahli, "Comparative Study of the Genetic Algorithms and Hessians Method for Minimization of the Electric Power Production Cost", International Journal of Computational Intelligence, Vol. 4, No. 3, pp. 217-224, 2008.
[6] L. Abdelmalek, M. Rahli, "Optimal Distribution of Active Powers by Hessian Methods", Journal Acta Electrotechnical, Vol. 49, No. 2, pp. 200-205, 2008.
[7] A. Belmadani, L. Benasla, M. Rahli, "A Fast Harmony Search Algorithm for solving Economic Dispatch Problem", Electrotechnical Review (Przeglad Elektrotechniczny), Vol. 86, No. 3, pp. 274-278, Warsaw, Poland, 2010.
[8] L. Abdelmalek, M. Rahli, "Comparative Study of Hessian Methods for the Optimization of Active Powers", Annals Computer Science, Tome 11, Fasc. 2, pp 61-66, Timisoara, Romanian, December 2013.
http://anale-informatica.tibiscus.ro
[9] A. Jafari, N.M. Tabatabaei, N.S. Boushehri, "Reactive Power Optimization using Intelligent Search Algorithms considering Voltage Stability Index", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 28, Vol. 8, No. 3, pp. 1-8, September 2016.
[10] M. Rahli, L. Benasla, A. Belmadani, L.A. Koridak, "Real Power-System Economic Dispatch Using a Variable Weights Linear Programming Method", Journal of Power Technologies, Vol. 95, No. 1, pp. 34-39, 2015.
[11] A. Lakhdar, "Comparative Study of Hessian Methods and Genetic Algorithms for Production Cost Minimization", Ph.D. Thesis, University of Sciences and Technology of Oran, pp. 53-63, Oran, Algeria, April 2008.
[12] R. Effatnejad, S. Bagheri, M. Farsijani, R. Talebi, "Economic Dispatch with Particle Swarm Optimization and Optimal Power Flow", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 14, Vol. 5, No. 1, pp. 9-16, March 2013.

BIOGRAPHIES

Name: **Abed**
Surname: **Bouadi**
Birthday: 04.08.1960
Birth Place: Relizane, Algeria
Bachelor: Electrotechnique, Electrical Department, Electrical Engineering Institute, University of Sciences and Technology of Oran, Oran, Algeria, 1987
Master: Power Electronics, Electrical Department, Electrical Engineering Institute, University of Sciences and Technology of Oran, Oran, Algeria, 2003
Doctorate: Contribution to Control of Inverters for Heating by Electromagnetic Induction, Department of Physic, Faculty of Physic, University of Sciences and Technology of Oran, Oran, Algeria, 2016
The Last Scientific Position: Assoc. Prof., Department of Physic, Faculty of Physic, University of Sciences and Technology of Oran, Oran, Algeria, Since 1987
Research Interests: Magnetic Materials, renewable energy, Power Systems
Scientific Publications: 30 Papers, 48 Master Supervision

Name: **Abdelkader**
Surname: **Mostefa**
Birthday: 02.11.1983
Birth Place: Relizane, Algeria
Bachelor: Education in Electronics, Department of Electrical Engineering, National Polytechnic School of Oran, Oran, Algeria, 2006
Master: Electrical Control, Department of Electrical Engineering, National Polytechnic School of Oran, Oran, Algeria, 2009
Doctorate: Renewable Energies, Department of Electrical Engineering, Faculty of Electrical Engineering, University of Science and Technology, Oran, Algeria, 2021
The Last Scientific Position: Assoc. Prof., Department of Electrotechnical and Automatic, Faculty of Sciences and Technology, University of Relizane, Relizane, Algeria, Since 2015
Research Interests: Renewable Energies, Electrical Control, Power Electronics, Power Systems
Scientific Publications: 3 Papers, 2 Projects

Name: **Mostefa**
Surname: **Rahli**
Birthday: 24.10.1949
Birth Place: Mascara, Algeria
Bachelor: Electro Technique, Department of Electro Technique, Faculty of Electrical Engineering, University of Sciences and Technology of Oran, Oran, Algeria, 1979
Master: Power systems, Department of Electro technique, Faculty of Electrical Engineering, University of Sciences and Technology of Oran, Oran, Algeria, 1985
Doctorate: Optimization in Power Systems, Department of Electro Technique, Faculty of Electrical Engineering, University of Sciences and Technology of Oran, Oran, Algeria, 1996
The Last Scientific Position: Prof., Department of Electro Technique, Faculty of Electrical Engineering, University of Sciences and Technology of Oran, Oran, Algeria, 1981-2018, Prof in Department of Electrotechnical and automatic, Faculty of Sciences and Technology, University of Relizane, Relizane, Algeria, Since 2018
Research Interests: Power Systems, Optimization, Artificial Intelligence
Scientific Publications: 180 Papers, 2 Books, 40 Projects: 40, 46 Theses