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ISSUE OF FLOW DISCHARGE IN MOTION OF MULTIPHASE FLUID WITH HYDRAULIC RESISTANCE

G.M. Salmanova [1,2] R.S. Akbarli [2]

1. *Department of Theoretical Mechanics and Continuum Mechanics, Baku State University, Baku, Azerbaijan*
gsm-1907@mail.ru
2. *Department of Mechanics, Azerbaijan University of Architecture and Construction, Baku, Azerbaijan*
akbarli89@mail.ru

Abstract- Mechanics of fluid-deformable tube system in conjunction with modern methods finds application in mathematical modeling of dynamic processes of blood circulation in vessels and organisms. The study of regularities of interaction of fluids with the body, the surface of which they wet in the process of flow, is carried out for various fluid models (viscous fluid, non-viscous fluid, vapor-liquid mixture, fluid with bubbles, fluid with solid particles etc.). On the other hand, when calculating losses in pipelines in which a real fluid with viscosity flows, it is necessary to take into account hydraulic losses, i.e. irreversibly lost part of energy. Calculation of hydraulic losses in pipelines is one of the main tasks of hydrodynamics. Hydraulic losses in the movement of real fluid are caused, firstly, by the manifestation of viscous forces in the fluid, i.e. friction losses, and secondly, by the presence of various regulating and measuring valves on the pipeline - the so-called local resistances - sections of the hydraulic network, where there is a change in the flow velocity in magnitude and/or direction. In this paper on the basis of the Rakhmatulin model, shock wave propagation in multiphase and deformable pipes is considered. In turbulent flow of subsonic flows, the equation of momentum, the law of conservation of masses and the equation of state of the medium are constructed. The law of mass flow rate variation depending on time and on the pipe, cross-section is obtained. An efficient analytical method has been developed to solve the problem.

Keywords: Multiphase Media, Mass Flow Rate, Live Section, Drag Coefficient, Hydraulic Radius of Section.

1. INTRODUCTION

In most cases, when conducting studies of the interaction of bodies with fluids, various simplifications and assumptions are adopted to obtain a "convenient" model for analyzing the mathematical model of the "body-liquid" system. Thus, it is accepted that if the body is elastic, the deformation of the supporting body is practically negligible. This makes it possible to study the resulting dynamic processes on the basis of linear differential equations of motion. In this case, the hypothesis of small wave-like motion of the fluid is accepted. As a result, the motion of the fluid phase leads to the application of linearized differential equations [1].

In [2] the first step in solving the inverse problem of controlling the flow of viscous incompressible fluid through a system of pipelines was made. According to a given flow rate at one of the outlets, the required pressure at the inlet is calculated and then the "pump" pressure is gradually brought to this level. The paper [3] describes a difference method for solving problems of Newtonian and non-Newtonian fluids flowing with a free surface, taking into account viscous dissipation, temperature dependence of rheological characteristics and capillary effects at the interface.

Methods for eliminating singularities in the mathematical formulation of problems related to the conditions on the line of three-phase gas-liquid-solid contact and rheological models of viscoelastic media are proposed. In [4] a parametric model for the study of vortex steady-state flow in such fittings as bends and tees was proposed. Water was considered as a pumped medium. It was established that the choice of the branch radius was determined by the requirement of minimum pressure drop at its outlet. The coupled problem of hydroelasticity for a circular tube of annular cross-section with an absolutely rigid inner cylinder and an elastic, geometrically irregular outer shell, freely supported at the ends of the tube, was obtained in [5].

Studies show that in thin-walled elastic structures, the force resulting from the interaction of a multiphase medium depends on the deformation of the structure [6, 8, 9]. The construction of a mathematical model and solution of a specific problem in hydro elastic systems consisting of a multiphase fluid and a deformable solid body is relevant in theory and application. Mathematical modeling of multiphase and multicomponent medium motion consists of the equations of mass balance, balance of momentum, law of energy conservation and second laws of thermodynamics for each phase and components. It is necessary to supplement the equations of the

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mathematical model with specific relations. These relations include mechanical, thermodynamic, sometimes electromagnetic, chemical and other properties of the environment. The multiphase nature of the medium complicates the study of hydrodynamic processes and motions occurring in this medium. This is more sharply manifested in the propagation of waves arising from vibrations and shocks. The study of the regularities of these processes is of exceptional importance in modern technologies for the creation of energy devices, as well as in the military, etc. in the development of new methods and the creation of scientific bases for their analysis. As a rule, the results of many sections of physics, including shock wave physics, gas dynamics, explosion physics, hydraulics, thermophysics, and filtration theory, are jointly discussed in studies conducted from the position of continuum mechanics in the hydrodynamics of heterogeneous media. In practical issues related to the interaction of elastic structures with fluid, in cases of considering the conditions of contact of engineering devices and structural elements with the external environment (liquid and gas), the technological conditions of operation and the requirements of building codes are taken as a basis.

The question of the flow of bubbly liquids is of great practical importance even without an explanation of the causes of bubble formation. However, the following can be briefly said about the process. When the pressure drop is less than the pressure in the saturated liquid the process of boiling in a liquid occurs. During the process in this part of the flow, the liquid vaporizes and voids begin to form in this part of the flow, and these voids are in the form of bubbles. The resulting bubble spaces are filled with air. Due to the physical and chemical effects that occur during this process, researchers became interested in the phenomenon of acoustic cavitation. When cavitation occurs, mechanical damage is observed on the surface of a solid body with liquid inside. If there is an increase in pressure during further flow, the formed voids disappear, which can be observed by sharp cracking sounds. When the carrier phase is a droplet liquid and the transported phase is air bubbles, at the same time, the process of cavitation phenomenon formation is facilitated when gases dissolved in water are transported together with water. The flow property in the part of the flow where the volume changes and cavitation occur at the same time is fundamentally different from the property in the other part. The properties of gas bubbles in liquids, the expansion, oscillation and "explosions" of bubbles, and the occurrence of many mechanical properties have been substantiated in theoretical and experimental studies [10, 11].

Mathematical modeling of motions and processes occurring in a continuum consisting of a fluid and a gas bubble, which is a heterogeneous medium, is based on the following two basic assumptions [12-14]:

- The diameter of the bubbles is many times greater than the molecular kinetic dimensions.
- The diameter of bubbles is smaller than the distance over which the macroscopic parameters of mixtures or phases change significantly. The size of bubbles in a liquid is much smaller than the specific wavelengths created in the mixture.

The first assumption allows us to use classical assumptions and equations of mechanics of the whole medium to describe the processes inside the bubble. The next assumption lays the basis for expressing wave propagation and other macroscopic phenomena in heterogeneous media. In general, heterogeneous media, including the hydrodynamics of bubble fluids, are described by a multidate model that takes into account the dynamic effects due to the difference in velocities of the dispersed phase and the fluid [14]:

- Each elementary volume contains spherically shaped bubbles with the same radius, and the volume concentration of bubbles present is negligible $(\alpha_2 << 1)$.
- The chaotic and internal (rotation and deformation) energy of bubbles, which are dispersed particles, is not taken into account.
- Bubble interactions and collisions within the fluid are not considered.
- The possibility of bubbles sticking together, i.e. coagulation, disintegration and formation of new bubble forms is not taken into account.

In addition to the above assumptions, the following assumptions are usually considered in the characterization of liquids with bubbles inside [14], [15]:

- The heat conduction property of fluid viscosity exists only in the interaction of phases, it does not manifest itself in the macroscopic phenomenon of changes in the quantity of motion and energy.

The density of gas is noticeably less than the density of liquid. α_1 and α_2 are volume concentration of liquid and gas; and n is number or numerical concentration of bubbles in a unit volume of bubbly fluid [14].

$$\alpha_2 = \frac{4}{3}\pi a^3 n\,,\ \alpha_1 = 1-\alpha_2$$

where, a is the radius of gas bubble. The reduced densities of phases determine the density of the bubbly fluid ρ as a whole, characterizing the masses of phases in the unit volume of the mixture [16].

The average mass of the liquid phase is constant [14]:

$$T_1 = T_0 = \text{const}\,(\rho_2 c_2 << \rho_1 c_1, \rho_2 << \rho_1 c_1 \approx c_2)$$

where, $\rho_i\ (i=1.2)$ is average concentration characterizing the phase, $\alpha_i\ (i=1.2)$ is volume concentration of the phase, and c_i is specific heat capacity of the phase at stable pressure.

Mathematical modeling of the problem of some boundary layer in gases with gas-liquid mixture and small solid particles has been carried out, the problem of dynamics of sound waves in tubes with elastic walls filled with bubble liquid has been investigated [17-19].

2. FORMULATION OF THE PROBLEM

In the presented work, the pulsating flow of multiphase fluid in a thin-walled elastic tube of circular cross-section is investigated. In thin-walled elastic structures, the force resulting from the interaction of liquid and gas media depends on the deformation of the structure. Since the strain propagation velocity in the shell walls is much higher than the wave propagation velocity in the fluid, the elastic waves around the circumference of the shell are many times ahead of the pressure front in the fluid. As a result, the shell is enveloped by the wave, when the normal stress along the circumference of the tube propagates through the entire shell. In order to determine the load exerted by the fluid on the shell, it is necessary to conduct simultaneous investigations with the shells themselves. In this case, the motion of the shell is expressed by the equations of elasticity theory, the motion of the fluid is expressed by the equations of hydromechanics, and the boundary conditions are taken as the usual boundary conditions for the shell.

On the basis of the single-velocity theory of multiphase media, the hydraulic shock during the movement of a mixture in a circular pipe is considered. Assume also that the mass average temperature of the mixture is constant. The cross-sectional area of the pipe changes, if the x-axis is directed along the pipe axis, then $R=R(x)$. The pressures of all phases of the multiphase media are considered to be coincident and equal to the pressure of the medium. In addition to this f_j is porosity, represents the volume fraction of the phases of the mixture will be [6]:

$$f_j = \frac{\rho_j}{\rho_j^0} = \frac{\text{reduce density of the } j\text{-th phase}}{\text{true densuty of the } j\text{-th phase}}$$

where, ρ_j is reduced density of the jth phase;

Then the density of the medium is defined by:

$$\rho = \rho_1 + \rho_2 + ... + \rho_n = \rho_1^0 f_1 + \rho_2^0 f_2 + ... + \rho_n^0 f_n$$

where, $\sum_{j=1}^{n} f_j = 1$, and $\rho_j^0 = \phi_j(p)$.

Let us introduce the speed of sound in the medium, which, due to the arbitrariness of the initial state, can be rewritten in the form:

$$c^{-2} = \rho_0 \sum_{j=1}^{n} \frac{f_j}{\rho_j^0 c_j^2}$$

However, if the pore pressure differs from the skeletal pressure, the formula is not applicable. It should be noted that, when compiling the equations of motion, the characteristics of fluid resistances established for stationary motions also take place for unsteady flow currents. In this case, the law of conservation of mass and equation of momentum will be written as [6]:

$$-S(x)\frac{\partial p}{\partial x} = \frac{\partial M}{\partial t} + M\frac{\lambda u}{8\delta} + \gamma S(x)\sin\alpha + \frac{\partial}{\partial x}\left[(1+\beta)M u\right] \quad (1)$$

$$-S(x)\frac{\partial p}{\partial t} = c^2 \frac{\partial M}{\partial x}$$

And so, we obtain a system of two first-order partial differential equations of hyperbolic type. When writing this system of equations, it should be taken into account that for the motion of a real fluid in the case of small subsonic velocities, convective terms can be neglected in the equations of motion. Where, α is elevation angle of the element axis dx above the horizon, for our problem $\alpha = 0$ and therefore $\gamma S(x)\sin\alpha=0$, and β is the Coriolis correction for non-uniform velocity distribution in the expression of the momentum of the flow in terms of the average velocity and the average density in the section.

In unsteady motion β will be a variable value depending on the nature of velocity distribution in the pipe section, where, M is mass flow rate, which depends on time and on x, $M = M(x,t)$, δ is hydraulic radius of the section, $S(x)$ is area of the "live" section, and λ is a drag coefficient.

In this case, the drag coefficient and Coriolis correction in each specific problem are subject to determination. We will assume that the drag coefficient for unsteady motion is the same function of the Reynolds number as for steady motion. We will study motion in long pipelines with subsonic velocity. For the flow of subsonic velocities in turbulent flow, let us average the term $\frac{\lambda u}{8\delta}$ as $\left[\frac{\lambda u}{8\delta}\right]_{av} = 4\frac{R}{\delta_0}$ [6].

Where δ_0 is pipe wall thickness. Solving the system (1) for subsonic velocities, when we can neglect the change of velocity heads $\beta=0$, we obtain:

$$\frac{\partial^2 M}{\partial t^2} + 4\frac{R(x)}{\delta_0}\frac{\partial M}{\partial t} + 16\frac{R^2(x)}{\delta_0 \lambda}\frac{\partial M}{\partial x} + 32\frac{M}{\lambda}R(x)\frac{\partial R(x)}{\partial x} - c^2\frac{\partial^2 M}{\partial x^2} + 2c^2\frac{1}{R(x)}\frac{\partial M}{\partial x}\frac{\partial R(x)}{\partial x} = 0 \quad (2)$$

3. RESULTS AND DISCUSSIONS

In the special case when the "live" section of the pipe is unchangeable along the x-axis, $R=const$ then integral Equation (2) allows us to determine the mass flow rate in a pipe of unchanged cross-section. In this case, Equation (2) takes the following form:

$$\frac{\partial^2 M}{\partial t^2} + 4\frac{R}{\delta_0}\frac{\partial M}{\partial t} + 16\frac{R^2}{\delta_0 \lambda}\frac{\partial M}{\partial x} - c^2\frac{\partial^2 M}{\partial x^2} = 0 \quad (3)$$

For this case the boundary and initial conditions are respectively given as follows:

$M(0, t) = 0$, $M(1, t) = M_0$; $M(x, 0) = 0$

The flow of a three-phase fluid consisting of air, light oil [9] and water in a cylindrical pipe of constant cross-section is investigated for numerical report in a special case. The volume fractions of the phases are respectively taken as 0.1; 0.3; 0.6. The radius of the pipe is taken as 0.05m and the wall thickness 0.003m. In case of turbulent movement of the working flow, the hydraulic resistance factor for a circular cross-section main with smooth surfaces is taken as 0.0256. We consider that small excitations caused by a monochromatic source occur in a hydro elastic system consisting of a thin infinite cylindrical tube of circular cross section containing a three-phase medium. We have taken the origin where the source is located and direct the x-axis along the pipe axis and study the change of flow rate per unit length of the pipe. The results obtained are presented in 3D graph format, Figures 1-3.

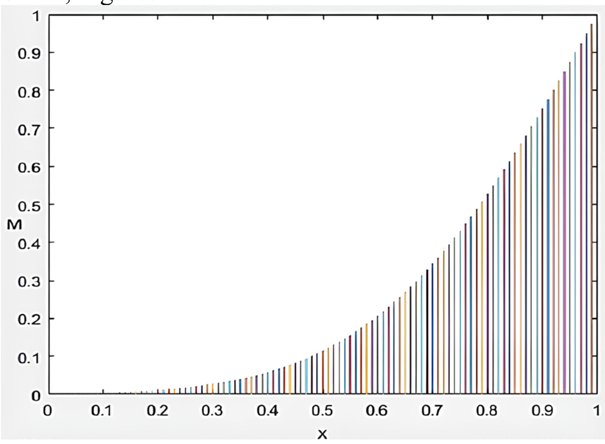

Figure 1. Variation of fluid flow rate along the axis of the tube

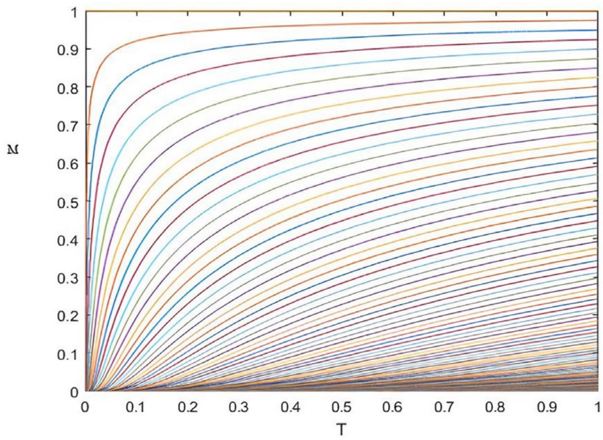

Figure 2. Change in fluid flow rate over time

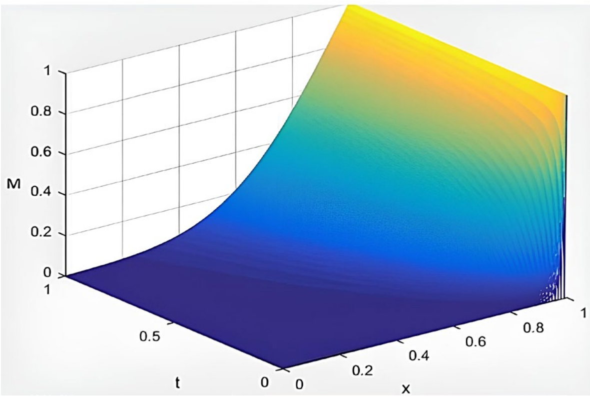

Figure 3. Dependence of flow rate on the x-coordinate directed along pipe axis and time

For comparison, in article [8] considered the issue of flow discharge in the motion of two-phase viscous fluid in deformable elastic shells without taking into account hydraulic resistance. An interesting point in this work is how the amplitudes of the characteristics found using the flow rate vary as a function of the number of bubbles per unit volume and the density of the fluid. For this purpose, the behavior of four liquid samples containing air bubbles (water, glycerol, ethanol and oil) was considered for numerical studies. It should be noted that the question is practically very important. Graphs of dependence of density, pressure and displacement amplitudes on the number of bubbles in a unit volume for various liquids were plotted Figures 4-6. At this time, the calculations were performed in the values α_{20}=0.01÷0.1.

Note that these calculations also considered different values of the parameter x; 0; 0.05; 0.1 and found that changing this parameter has little effect on the amplitudes. The displacement decreases as a function of the number of bubbles, a decrease by about 2 times for each of the fluids considered. The change in the amplitude of the displacement as a function of the density of the carrier liquid increases by about 1.39 times in the case of glycerol and oil with increasing density.

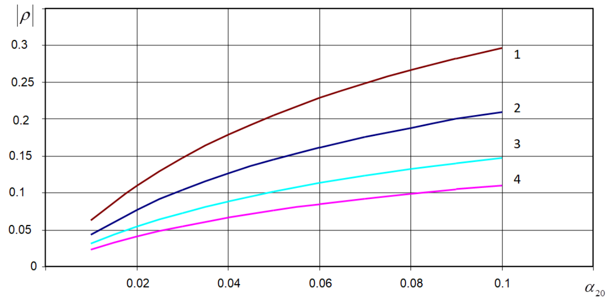

Figure 4. Dependence of the density of two-phase fluid on the number of bubbles; 1) glycerol, 2) water, 3) ethanol, 4) light oil

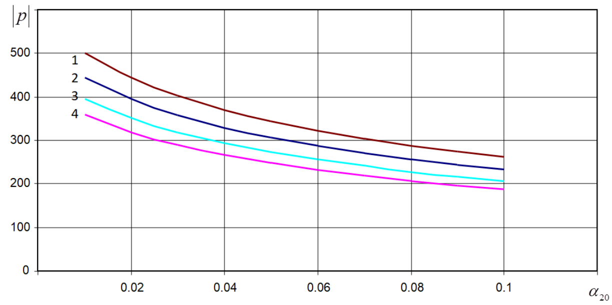

Figure 5. Dependences of the hydrodynamic pressure of a two-phase fluid on the number of bubbles; 1) glycerol, 2) water, 3) ethanol, 4) light oil

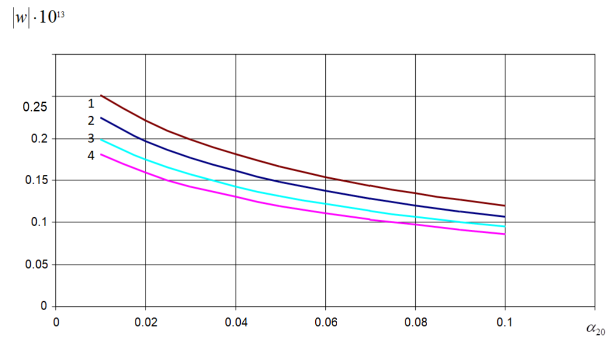

Figure 6. Dependence of displacement on the number of bubbles during the movement of two-phase fluid; 1) glycerol, 2) water, 3) ethanol, 4) light oil

4. CONCLUSIONS

The paper considers a specific problem of motion of three incompressible and compressible media in pipes, on the basis of which the regularities of flow of mixtures are revealed. An effective numerical method for the calculation of real physical natural processes and techniques is created. The developed mathematical models of motion of multiphase media and their solutions can find applications in the creation of biomechanical theory, in solving the modern problem of gas and oil mechanics and in the transportation of mixtures. Knowing the law of variation of the "live" section of the pipe, we can specify by using the integral Equation (2) the changes of mass flow rate as a function of time and along the pipe axis. The equation is solved by combining the boundary and initial conditions.

Numerical results are performed for a pipe of constant cross-section in which real multiphase fluid flows. The flow rate variation depending on the pipe axis and in the unsteady case on time is shown in the graphs. Reports are made for turbulent flows considering hydraulic resistance in a pipe with a smooth inner surface. It is shown that the multiphase property is fundamentally different from the effect of wave propagation in a tube filled with a single-phase fluid. Although this feature complicates the mathematical solution of this problem, it plays an important role in the study of dynamic processes.

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BIOGRAPHIES

Name: **Gulnar**
Middle Name: **Musa**
Surname: **Salmanova**
Birthday: 30.08.1982
Birthplace: Lachin, Azerbaijan
Bachelor: Mechanics, Faculty of Mathematics-Mechanics, Baku State University, Baku, Azerbaijan, 2003
Master: Fluid Mechanics, Faculty of Mathematics-Mechanics, Baku State University, Baku, Azerbaijan, 2005
Doctorate: Fluid Mechanics, Faculty of Mathematics-Mechanics, Baku State University, Baku, Azerbaijan,

2012
The Last Scientific Position: Assoc. Prof., Department of Theoretical Mechanics and Continuum Mechanics, Baku State University, Baku, Azerbaijan, Since 2012
Research Interests: Theoretical Mechanics, Hydromechanics, Solid State Mechanics and Mechanics of Composite Materials
Scientific Publications: 25 Papers, 3 Books, 36 Theses

Name: **Reyhan**
Middle Name: **Sayyad**
Surname: **Akbarli**
Birthday: 18.07.1989
Birthplace: Neftchala, Azerbaijan
Bachelor: Mechanics, Faculty of Mathematics-Mechanics, Baku State University, Baku, Azerbaijan, 2010
Master: Fluid Mechanics, Faculty of Mathematics-Mechanics, Baku State University, Baku, Azerbaijan, 2012
Doctorate: Fluid Mechanics, Faculty of Mathematics-Mechanics, Baku State University, Baku, Azerbaijan, 2022
The Last Scientific Position: Senior Lecturer, Department of Mechanics, Azerbaijan University of Architecture and Construction, Baku, Azerbaijan, Since 2012
Research Interests: Theoretical Mechanics, Hydromechanics, Solid State Mechanics and Mechanics of Composite Materials
Scientific Publications: 18 Papers, 1 Book, 25 Theses