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ATTITUDES ON MODELLING DIFFICULTIES WITH DIFFERENTIAL EQUATIONS

Y. Naamaoui [1] M. Chergui [2] B. El Wahbi [1]

1. Faculty of Science, Ibn Tofail University, Kenitra, Morocco, yassine.naamaoui@uit.ac.ma, bouazza.elwahbi@uit.ac.ma
2. Regional Centre for Education and Training Professions, Kenitra, Morocco, chergui_m@yahoo.fr

Abstract- The teaching of mathematical modeling is a new trend in mathematics teaching practices. It has the advantage of confronting real-life situations and the mathematical field, in a more advanced way than that practiced in problem-based teaching. This has the advantage of dispelling any stereotypes about a disconnection between the two fields. But the practice of teaching modeling is still quite difficult for teachers, for several conceptual and institutional reasons. In this study, we explore the attitudes of mathematics teachers to difficulties that secondary school students may encounter in the mathematization, working mathematically and interpretation stages of the differential equation modeling process. To this end, we interviewed a randomly selected group of secondary school teachers who had actually taught differential equations. We deduced that the technical aspect of solving differential equations is seen as the main obstacle to implementing the modeling process. The second factor marking the teachers' attitude is that they are convinced that the lack of understanding of the situation that leads to a differential equation from a conceptual point of view has a negative impact on the satisfactory progress of the modeling stages. It should also be noted that the teachers surveyed were neutral towards certain types of difficulty.

Keywords: Modeling, Modeling Cycle, Difficulties in Modeling, Attitudes, Differential equation.

1. INTRODUCTION

Teaching and learning mathematics by/for modeling has become an important trend in educational practices. This is due to the various positive impacts of this approach[1]. At first, modeling was investigated in solving problems from real life, like describing how phenomena evolve. Conversely, the development of mathematical models has made it possible to better understand astronomical movements, physics situations, and conjecture the future state of random phenomena. Despite this distinguished place for mathematical models, modeling was not extensively included in mathematical instruction, and the focus was mostly on solving abstract mathematical problems. The integration of modeling into teaching mathematics is a result of contemporary pedagogical paradigms, particularly the emergence of constructivism and its educational uses.

The integration of modeling in education is also due to several factors that promote its adoption. For example, modeling encourages multidisciplinary, and technological advances have led to the expansion of modeling and simulation software that makes it easier for teachers and students to create, manipulate, and visualize models. The integration of modeling, on the other hand, has been encouraged by educational reforms that aim to support student-centered pedagogical approaches. By involving students in thinking and problem-solving activities, modeling provides an opportunity to helping them develop these skills and shows them how theoretical concepts are applied in real-world situations.

One of the oldest and most common mathematical tools in modeling is differential equations (DEs) [2]. They have long been included in mathematics curricula. Martinez-Luaces [3], for instance, used differential equation modeling exercises in a first-year chemical engineering and food technology engineering course to look into significant questions about the stability, uniqueness, and existence of mixing problems in instructional experiments. He noticed that the students had an encouraging reaction and were motivated to collaborate on projects to identify the best model for the situation. In order to teach differential equations to two groups of undergraduate engineers, Czocher [4] tried comparing two pedagogical approaches: one used decontextualized situation for solving DEs, and the other used modeling principles to understand differential equations as real-world models. She realized that, statistically, the second approach improved student learning.

The impact of including context into differential equation instruction on engineering students' capacity to create and comprehend differential equations was examined by Sijmkens et al [5]. Whereas the majority of the problems given to the control group missed context and concentrated only on procedural knowledge and skills, the experimental group was given context-rich problems where the emphasis was on creating a differential equation and interpreting its solution in the given context. They observed that providing students with context-rich problems improved their ability to construct and

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understand differential equations. It also turns out that procedural knowledge remains intact in the process of students becoming more proficient in these skills.

Nevertheless, a number of studies have indicated that the way they are currently taught is unsatisfactory. In this regard, Blum [6] emphasized some types of obstacles in implementing modeling in the classes. Organizational obstacles manifested by the limited time devoted to modeling in the classroom. Schmidt [7] also evoked this constraint. On the other hand, some teachers criticize the overloading of the mathematics curriculum, and others even doubt that applications and links with other disciplines have a place in mathematics teaching, as these elements tend to disrupt the clarity and simplicity of the mathematical context [8]. The second type is linked to the obstacles encountered by students. Modeling brings too many difficulties to the course and makes it less comprehensible for students, who may have difficulty with certain steps or even with the whole modeling process [9]. Furthermore, compared to traditional mathematics courses, where mathematical tasks are more familiar to many students because they are much easier to understand and can often be completed by simple activities, like calculations, which make students feel more successful, problem solving, modeling, and applications to other disciplines make mathematics courses more complex and less predictable for learners [8].

In a study aiming to understand the difficulties in the modeling process, M. Anaya, et al. [10] conducted a study with a group of engineering students. At the mathematization stage, a large number of students used proportionality and some algebraic tools to describe the situation presented to them, and no differential equation was stated to describe the process. The mathematical knowledge they involved did not correspond to that studied at the university. For another group, the students were guided through the mathematization process to obtain a first-order linear differential equation. However, the authors observed that 80% of the students in this group showed difficulties in solving the differential equation. These difficulties were mainly linked to the recognition of the type of differential equation and procedural difficulties linked to integrals and functions.

Rasmussen and Marrongelle [11] have noted that students have conceptual difficulties in distinguishing between the rate of change of a quantity and quantity in the context of a differential equation modeling activity. Difficulties are not limited to learners; researchers have also noted difficulties among teachers when modeling. In this context, let us recall that Borromeo Ferri [12] observed three distinct categories of teachers. Some teachers focused more on formal aspects while providing assistance to students in their modeling process and discussions, while others focused on concrete aspects to validate results and help students. A final type of teacher took both formal mathematical and real-world aspects into consideration.

For example, Ng [13] found that pre-service teachers had difficulties appreciating the constructive effects of modeling activities due to insufficient experience with this type of task or with the required type of thinking for mathematical modeling in their mathematical training.

Through the use of a modeling module in the training program, Anhalt and Cortez [14] studied how student teachers who had never taken courses on mathematical modeling evolved in their understanding of the subject. They stated that these teachers were able to acquire an accurate comprehension of modeling as an iterative process that involves the formulation of theories and the verification of conclusions pertaining to situations in the real world. One of the challenges associated with teaching is that it takes time for teachers to prepare and modify modeling scenarios to fit the level of the class they are teaching. Additionally, as more "non-mathematical" concepts are added to the teaching of mathematics and make learning assessment more difficult, problem solving and connections to the world other than mathematics open teaching to other demands for teachers. Some teachers even find it difficult to deal with examples from subjects they haven't studied, which makes it challenging to adapt examples of applications and modeling for the classroom [8].

In Addition, Asempapa and Sastry's [15] study of pre-service teachers' knowledge and practices concerning mathematics revealed that their knowledge and experience appear limited, and their understanding of modeling practices is traditional. Other difficulties are noticed; they have confused mathematical modeling with mathematical models which seems to be the result of the absence of formal training in the practices associated with modeling.

Sen Zeytun, et al.'s investigation [16] concentrated on identifying the causes of pre-service teachers' challenges with mathematical modeling. They demonstrated how contextual and individual factors make up the sources of difficulties. Individual factors include a preference for focusing only on obtaining a result, a lack of conceptual understanding of mathematics, difficulty connecting the mathematical realm to the real world, and unorganized, nonsystematic work. Contextual elements include time constraints and a lack of prior modeling assignment experience.

After investigating the attitudes of physics teachers toward differential equation modeling [17]; in the present work we aim to explore the attitudes of mathematics teachers toward the learning difficulties of differential equation modeling linked to the steps mathematizing, working mathematically, and interpreting. This objective can be expressed by the following central research question: what are the attitudes of in-service mathematics teachers towards the difficulties encountered during mathematizing, working mathematically, and interpreting in differential equation modeling?

This exploration is motivated by a concern to improve our understanding of teachers' representations of the DE modelling process, to be subsequently exploited in determining efficient interventions when implementing the various stages of modelling in classrooms.

2. THEORITICAL FRAMEWORK

Mathematical modeling generally means the process of trying to find a mathematical explanation for a non-mathematical object or issue. The situation to be modeled and the mathematical framework are two entities that are linked structurally through the modeling process. In other words, in a mathematical modeling exercise, we apply mathematics to solve a non-mathematical problem. An original situation (problem), a final desired situation (which represents a solution to the original situation), and a collection of modeling tools can all be used to characterize a mathematical modeling activity. Mathematical models are conceptual systems that are used to describe or explain the behavior of other systems, according to Lesh and Doerr [18]. Systems of external notation are used to express them.

According to Blum and Niss [8], modeling is a process made up of steps that make up a modeling cycle. Depending on the investment environment, authors formulate this cycle in different ways. Mathematical modeling competency is defined by Niss, et al. [19] as the capacity to recognize pertinent questions, variables, relations, or assumptions in a given real-world situation, to translate these into mathematical form, and to interpret and validate the mathematical problem's solution in relation to the situation. It also includes the capacity to analyze or compare given models by examining the underlying assumptions and verifying the models' properties.

Blum [20] claims that modeling competence is a multifaceted concept that is made up of various sub-competences. One of the most used modeling cycles is that of Blum and Leib [21], which consists of the following seven steps, also referred to as sub-competencies:

- Understanding: converting the situation studied into mental images.
- Simplifying: decomposing the situation into understandable elements.
- Mathematizing: converting the situation studied into mathematical objects.
- Working mathematically: applying mathematical skills and heuristic approaches to address the mathematical problem.
- Interpreting: determining the meaning of mathematical results obtained concerning real-life data.
- Validating: approving the correspondence between real-life situations and the mathematical model.

In this paper, we are interested in exploring the representations of teachers with respect to the three sub-competencies, mathematizing, working mathematically and interpreting in a modelling process with DEs. Greefrath and Vorholter [22] state that students will have mastered these sub-skills when they can effectively convert appropriately simplified real-life situations into differential equations, apply heuristic techniques and mathematical knowledge to solve differential equations, and apply the model's results to the real world in order to produce real outcomes.

Borromeo Ferri and Blum [23] have characterized the various components of competence for teaching mathematical modeling as follows: the theoretical component, which covers knowledge of modeling cycles and their relevance to different objectives, and the task-related component, which includes knowledge of and the aptitude for approaching modeling tasks in different ways. This enables the teacher to identify different approaches to the task and thus gain an insight into the diversity of solutions. The pedagogical component is characterized by the selection of an adequate learning context. Finally, the diagnostic component concerns the categorizing of students' modeling activities in the different phases of the modeling process, and the identification of any cognitive obstacles in the process.

It is interesting to note, in this context, that the work of making explicit the resources that are necessary for effective modeling teaching fits into a more general framework of conceptualizing the professional competencies of mathematics teachers initiated by Shulman [24]. Baumert and Kunter [25] claim that professional competence consists of various elements, including specific declarative and procedural knowledge, motivational orientations, values, beliefs, and professional goals, as well as professional self-regulation skills.

The distinction and transition between real models and mathematical models are difficult during mathematization, which is an obstacle for learners, especially as it requires basic mathematical knowledge. Furthermore, [27] have tried to study the obstacles encountered by science students when modeling, they identified blockages in the modeling process: for example, the choice of problem statement caused by poor student comprehension (forgetting essential data in the problem text, guidelines), and they observed the inability to convert specifications between variables and lack of algebraic knowledge.

Table 1. Difficulties in the modelling process by DEs

Modeling step	Difficulty categories
Forming a differential equation	M1: Failing to define the differential equation's variables
	M2: Failing to recognize the relationships among the variables in the differential equation
	M3: Failing to utilize suitable mathematical techniques while mathematizing
	M4: Failing to solve differential equations
	M5: Failing to understand the situation that leads to a differential equation from a conceptual point of view
	M6: Failing to understand mathematical concepts (symbols, signs, etc.) or differential equations.
Solving differential equation	Wm1: Failing to solve differential equations using sufficient formulas
	Wm2: Failing to utilize appropriate solution techniques and algorithms to find the general solution of a differential equation
	Wm3: Failing to solve differential equations using technology
	Wm4: Failing to understand the situation that leads to a differential equation from a conceptual point of view
	Wm5: Failing to solve the differential equation because it is too complicated
	Wm6: Failing to convert the units used in the differential equation
Interpreting	I1: Failing to recognize the accurate interpretation of some mathematical results
	I2: Inability to use mathematical results to respond to the issue

Several studies have been devoted to errors and difficulties in the modeling process, and [28] for example. By transposing the synthesis of these studies provided Klock and Siller [29] to the formation of a mathematical model and working mathematically adapted to the concept of differential equations, we obtain a list of difficulties related to each targeted modelling step. Table 1 presents the details of this transposition.

3. METHODOLOGY

This research is of an explorative character and aims to present a description, as comprehensive as possible, of the attitudes of mathematics teachers toward the difficulties of modeling differential equations. A mixed approach is therefore used, combining a qualitative analysis supported by the literature review with a quantitative analysis using statistical tools. For this purpose, we opted to survey mathematics teachers who actually teach in secondary school science or technology classes. In these classes, mathematics curricula stipulate that modeling with differential equations is a principal competence that students are required to acquire [30].

The survey is carried out using a questionnaire (Table 3), developed by considering the following hypotheses:
1. The teachers do not have the same representations on the modeling steps using differential equations.
2. The teachers do not have the same appreciation of the difficulties encountered by the students during the implementation of the modeling. The questionnaire items are exactly the same as those described in the previous section (Table 1). In order to validate the content of the questionnaire, it was presented to experts in the field of teaching mathematics and then administered to five mathematics teachers working in different secondary schools, who had previously worked as a secondary school final year teacher, and were randomly selected from among volunteers and not belonging to the study sample.

As a result of this pre-test, small adjustments in the wording of some questions were made to the questionnaire, which was then administered online in May 2023. The questionnaire was distributed online, with the help of educational inspectors, to groups of teachers working in several secondary schools in three regional education and training academies in Morocco.

Besides optimizing time and effort, the choice of administering the questionnaire online was motivated by the advantage of allowing respondents to remain anonymous. This reinforces the respondents' confidence that they will not be subjected to direct judgment as a result of their answers. The number of teachers who responded to the questionnaire was 40. We emphasize that data collection was restricted to teachers who declared themselves familiar with the modeling process. The participants' professional data on seniority in the profession and the number of years they had been teaching final year classes are described in Table 2.

An important factor in questionnaire quality was taken into account in the design of our methodological protocol. It concerns objectivity, which refers to the level of independency of test outcomes from any extraneous effects on respondents. Literature distinguishes between three types of objectivity: objectivity of implementation, objectivity of evaluation and objectivity of interpretation. To guarantee objectivity of implementation, it is necessary to minimize the test administrator's interaction with the respondents. Objectivity of evaluation is understood as the independency of the test evaluation from the respondent, in order to minimize the subjectivity of the answers collected. For this, the choice of closed questions, where participants have to choose between alternative predefined answers, is appropriate. Objectivity of interpretation means the independence of the interpretation of test results from the person analyzing them, which can be guaranteed by using a fixed scale to assign numerical values to each participant's responses.

Table 2. Professional characteristics of the sample

	Time spent as a teacher (by years)			Number of school years as a teacher of final scientific classes		
	Less than 5	Between 5 and 10	More than 10	Less than 3	Between 3 and 5	More than 5
Number	5	18	17	13	3	24

Table 3. Questionnaire on modeling difficulties using differential equations for high-school mathematics teachers

Time spent as a teacher	Less than 5 years		Between 5 and 10 years		More than 10 years	
Number of school years as a teacher of final scientific classes	Less than 3 years		More than 3 and less than 5		More than 5 years	
Possible difficulties linked to each step of the Modeling	Strongly disagree	Disagree	Neutral	Agree	Strongly Agree	
Forming a mathematical model						
M1						
M2						
M3						
M4						
M5						
M6						
Working mathematically						
Wm1						
Wm2						
Wm3						
Wm4						
Wm5						
Wm6						
Interpreting						
I1						
I2						

On the basis of this review, each survey participant was asked to respond to the items in Table 1 coded on a numerical scale from 1 to 5, varying from "Strongly disagree" to "Strongly agree", which corresponds to a Likert scale commonly used in research to measure attitudes and cognitive constructs. The choice of an odd-numbered scale has the advantage of providing the respondent with the possibility of positioning himself on a central response modality.

The detailed table of responses recapitulates all the coded answers for each teacher to the questions listed in the columns. The aim is then to analyze this table to extract the main characteristics in terms of attitudes that emerge from all the declarations collected. Considering the variables in our questionnaire, which are all qualitative with five modalities, Multiple Correspondence Analysis (MCA) is the most appropriate technique for analyzing and interpreting participants' responses with respect to the connections between the different modalities. Statistical analysis is undertaken using SPSS software.

4. RESULTS

Univariate analysis of the responses obtained yielded the results summarized in Table 4.

Table 4. Univariate results

	Strongly Agree	Agree	Neutrally	Disagree	Strongly Disagree
M1	7.5%	30%	10%	25%	27.5%
M2	5%	40%	17.5%	22.5%	15%
M3	17.5%	32.5%	12.5%	20%	17.5%
M4	22.5%	20%	15%	25%	17.5%
M5	20%	30%	15%	20%	15%
M6	12.5%	32.5%	22.5%	20%	12.5%
Wm1	12.5%	25%	12.5%	20%	30%
Wm2	15%	30%	7.5%	30%	17.5%
Wm3	15%	22.5%	20%	25%	17.5%
Wm4	12.5%	25%	7.5%	30%	25%
Wm5	15%	30%	10%	27.5%	17.5%
Wm6	0%	20%	32.5%	27.5%	20%
I1	22.5%	35%	12.5%	25%	5%
I2	17.5%	20%	20%	37.5%	5%

These preliminary results enable us to identify a number of conclusions. For example, for each of the modelling sub-steps considered, it is possible to identify the most dominant teacher attitude. In particular, difficulties in the modeling process where teachers strongly agree or strongly disagree can simply be discerned from these univariate statistics.

However, to give more credibility to this investigation, it is necessary to undertake a cross-study between the variables in terms of correlation. This will be carried out by an MCA. The first result to pay attention to is Cronbach's alpha, which indicates the internal consistency of the questionnaire. A measure is said to be reliable when it describes accurately the behavioral trait being measured. The MCA applied to the complete disjunctive table has shown that the number of dimensions (or axis) to be retained is 2. The results relating to reliability and the amount of variance that can be explained by each retained dimension are shown in Table 5.

Table 1. Model summary

Dimension	Cronbach's Alpha	% of Variance
1	0.947	59.246
2	0.867	36.679

We can regard our questionnaire as reliable, since the value obtained is greater than or equal to 0.7, the acceptable level in the field of educational research. The cross-tabulation of variables revealed significant correlations at the 0.05 level (two-tailed), as shown in Table 6.

Table 6. Correlations between variables

	M1	M2	M3	M4	M5	M6	Wm1	Wm2	Wm3	Wm4	Wm5	Wm6	I1	I2
M1	1													
M2	0.656	1												
M3	0.542	0.561	1											
M4	0.447	0.569	0.556	1										
M5	0.610	.655	.663	.763	1									
M6	0.503	0.668	0.615	0.612	0.732	1								
Wm1	0.434	0.500	0.586	0.463	0.595	0.492	1							
Wm2	0.512	0.619	0.647	0.540	0.661	0.616	0.712	1						
Wm3	0.507	0.518	0.534	0.882	0.695	0.551	0.484	0.617	1					
Wm4	0.491	0.373	0.444	0.503	0.598	0.249	0.566	0.606	0.571	1				
Wm5	0.408	0.475	0.717	0.572	0.644	0.432	0.647	0.727	0.635	0.696	1			
Wm6	0.380	0.477	0.665	0.491	0.495	0.517	0.648	0.625	0.444	0.483	0.513	1		
I1	0.517	0.453	0.531	0.299	0.487	0.313	0.549	0.535	0.450	0.558	0.613	0.506	1	
I2	0.573	0.534	0.516	0.507	0.647	0.544	0.603	0.720	0.631	0.581	0.653	0.452	0.615	1

Table 7. Discrimination measures

	Dimension 1	Dimension 2
M1	0.489	0.126
M2	0.559	0.128
M3	0.637	0.337
M4	0.587	0.673
M5	0.745	0.685
M6	0.534	0.335
Wm1	0.592	0.376
Wm2	0.728	0.267
Wm3	0.632	0.484
Wm4	0.513	0.562
Wm5	0.666	0.245
Wm6	0.506	0.415
I1	0.468	0.212
I2	0.638	0.289

The first point to note from these results is that there is no pair of variables with zero correlation. This confirms the relevance of our questionnaire as a tool for exploring the attitudes of mathematics teachers towards EDs modelling difficulties. As previously indicated, the MCA carried out revealed that it is possible to retain two main dimensions, dimension 1 and dimension 2, which have Cronbach's alpha values of 0.947 and 0.867 respectively. The discriminative values of each variable with respect to the two retained dimensions are presented in Table 7.

Figure 1 shows the representation of the different variables in the factorial plane formed by the two dimensions 1 and 2.

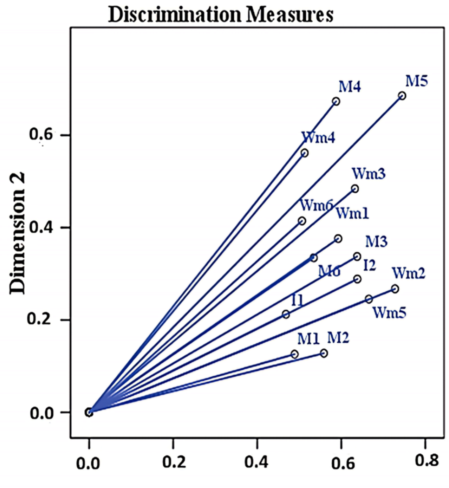

Figure 1. Representation of variables in the factorial plane

Figure 1 shows that the modalities are close to each other. This means that they are well correlated, but not too strongly. This can be explained by the fact that there is no perfect similarity in the responses of the teachers surveyed. Consequently, it is possible to consider that their representations of the steps involved in modeling with differential equations are not the same, and that they do not have the same appreciation of the difficulties encountered by students when implementing modeling.

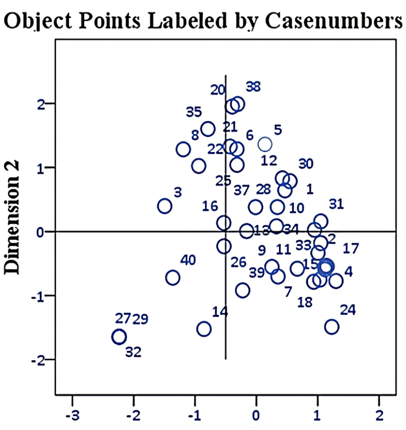

Figure 2. Representation of subjects

This fact can be clearly seen in in Figure 2, where the representation of individuals in the plane formed by the two main dimensions resulting from the MCA is illustrated.

5. DISCUSSION

With respect to mathematization in the differential equation modeling process, we note from the data in Table 1 that the group of teachers surveyed are equally divided between those who see failing to define the differential equation's variables as a difficulty for students and those who see the opposite. This first result seems a little surprising. Let us use reasoning by the absurd to justify our judgement. In fact, if we assume that this M1 difficulty does not arise, this directly implies that the steps of converting the situation studied into mental images and decomposing the situation into understandable elements can be performed by secondary school students without any problems, so that they can identify the variables they need to involve correctly. However, most studies show quite the opposite. This contradiction can be explained, in the light of Borromeo Ferri and Blum's [23] work, by the fact that theoretical and task-related dimensions of competence for teaching mathematical modeling are not well developed among the group of teachers.

In this mathematization stage, we note that teachers are most determined on three types of difficulties. The first is failing to solve differential equations, followed by failing to understand the situation that leads to a differential equation from a conceptual point of view, and finally failing to utilize suitable mathematical techniques while mathematizing. In the working mathematically stage, a large proportion of teachers disagree that failing to understand the situation that leads to a differential equation from a conceptual point of view is a difficulty for students. It is a conclusion that is in line with the contradiction we mentioned earlier. The second significant aspect of this stage is that 50% of teachers disagree that solving differential equations using sufficient formulas hinders modelling. For failing to use appropriate solution techniques and algorithms to find the general solution of a differential equation or to solve it because it is too complicated, we note that teachers are almost equally divided between those who agree and those who disagree about the place of this difficulty. The proportion of teachers who agreed with the difficulty failing to convert the units used in the differential equation achieved the lowest score, and a third of teachers were undecided.

Regarding the interpretation of the findings, teachers are more persuaded that a modeling difficulty emerges when certain components of the mathematical results are not correctly understood. On the other hand, the results do not reveal any clear trend. This multitude of deductions from the univariate study does not allow us to identify the main characteristics of teachers' attitudes. Added to this is the fact that in several cases of difficulty, teachers remained undecided at a considerable rate. It is therefore necessary to advance our analysis of the results by conducting a cross-reading of the variables involved.

Table 3 shows that the teachers' responses confirm positive and generally medium correlations. Some strong correlations between difficulties proposed in the questionnaire prove a coherence in the teachers' answers, as for example, attitudes on difficulties M4 and Wm3, where the correlation coefficient indicates 0.882. It could even be said that some of these strong correlations are justified from a cognitive point of view. In this respect, we cite the examples of the positive correlation between failing to understand the situation that leads to a differential equation from a conceptual point of view and failing to solve differential equations, and between failing to solve differential equations using sufficient formulas and failing to utilize appropriate solution techniques and algorithms to find the general solution of a differential equation. This is also the case for difficulties Wm2 and Wm5. The lowest correlations are recorded between difficulty types M6 and Wm4 on the one hand, and between I1 and M4 on the other.

To offer a more extensive comprehension of the different attitudes of mathematics teachers towards the difficulties in implementing modelling steps by DEs, as described in the preceding passages, it is essential to identify the main dimensions associated with these attitudes, based on the multiple correspondence analysis (MCA). To achieve this, we first have to determine each of the two dimensions derived from the MCA. Figure 1 indicates that the variables illustrated by questions Wm2, Wm5 and I2 are best represented in an ordered way with respect to the first dimension. Figure 1 indicates that the variables illustrated by the difficulties Wm2, Wm5 and I2 are best represented in an ordered way with respect to the first dimension. It should also be noted that these difficulties are highly correlated with each other.

This first main trait in teachers' attitudes means that they consider the difficulty of solving a differential equation and the failure to provide a response to the question using the mathematical tools to form a real obstacle in the development of DE modeling skills in students. The second dimension deduced from the MCA is fairly well characterized by difficulties M4 and M5, which relate to the mathematization stage in the modeling process. It should also be emphasized that failing to understand the situation that leads to a differential equation from a conceptual point of view is the most well represented difficulty in this axis. In other words, this axis reflects awareness of the importance of identifying situations that require modeling by EDs.

Taking these two dimensions into account, and reading Figure 2, we can deduce that a large proportion of the teachers surveyed fall positively on the first factorial axis. This means that the majority of teachers share the attitude that failure to solve differential equations is an obstacle to modelling in learners. The consistency shown by teachers in this attitude no longer persists in the case of Axis 2. In fact, the survey population is divided into two groups, with opposite responses.

6. CONCLUSION

Modeling is a very important process in and for learning mathematics. Implementing it in the classroom is not a simple task. There are several reasons for this. Firstly, there are factors that can be described as conceptual, others linked to the implementation of the steps and then the pedagogical ones. It is clear, then, that any attempt to attenuate the impact of these factors can only be to the benefit of learning modeling or using it to develop specific mathematical skills. This motivated us to undertake this study to explore the attitudes of mathematics teachers to the difficulties that secondary school students may encounter in modeling using differential equations. The choice of this mathematical concept is justified by its major usefulness in the study of many phenomena from many fields.

We focused on the following three stages of the modeling process: mathematization, working mathematically and interpretation. These steps are very important. In this context, we would point out that the training of future engineers requires a good command of the skills of mathematizing, appealing to the appropriate mathematical tools and manipulating them easily, and above all being able to interpret the results obtained. For each of these steps, we drew up a list of difficulties based on the literature review we had conducted. This list formed the essence of the items in a questionnaire we administered online to a group of teachers, all of whom had experience in teaching differential equations at secondary school.

Analysis of the main results we have deduced has enabled us to observe particular attitudes on the part of teachers. First of all, we noted a neutrality on the part of teachers towards several types of difficulties that students may encounter when modeling with differential equations, such as the importance of converting the units used in the differential equation. This lack of decision-making on the part of teachers reflects a lack of conceptual mastery of the notion of modeling. Added to this is the fact that, for other difficulties, the group of teachers surveyed is evenly divided into opposing positions. From the MCA undertaken, we have deduced that the technical aspect in solving DEs is considered to be the main factor hindering the implementation of the modeling process. The second factor marking teachers' attitudes is that they are convinced that a lack of understanding the situation that leads to a differential equation from a conceptual point of view has a negative impact on the good progress of modeling steps by DEs.

At the end of this paper, we consider that the current work is an added value on the question of learning and teaching modeling by differential equations. Indeed, the results obtained can be exploited to set up a training program for future mathematics teachers in teaching modeling by differential equations, or to animate continued training workshops for in-service teachers. In fact, the present study offers insights for regulating teachers' misconceptions and overcoming institutional modeling difficulties.

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BIOGRAPHIES

Name: **Yassine**
Surname: **Naamaoui**
Birthday: 22.01.1985
Birthplace: Kenitra, Morocco
Bachelor: Applied Mathematics, Department of Mathematics, Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, 2012
Master: Didactics, Teaching and Training Professions, Department of Mathematics, Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, 2018
Doctorate: Student, Department Mathematics, Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, Since 2018
The Last Scientific Position: Teacher of Mathematics, Secondary School, Regional Academy of Education and Training, Rabat-Sale-Kenitra, Kenitra, Morocco, Since 2014
Research Interests: Didactic of Mathematics, Didactics of Sciences, Applied Mathematics
Scientific Publications: 2 Papers, 5 Communications

Name: **Mohamed**
Surname: **Chergui**
Birthday: 09.12.1969
Birthplace: Larache, Morocco
Bachelor: Mathematics, Statistics, Department of Mathematics, Faculty of Sciences, Abdelmalek Essaadi University, Tetouan, Morocco, 1991
Master: Mathematics, Functional Analysis, Department of Mathematics, Faculty of Sciences, Moulay Ismail University, Meknes, Morocco, 1998
Doctorate: Applied Functional Analysis, Department of Mathematics, Faculty of Sciences, Moulay Ismail University, Meknes, Morocco, 2009
The Last Scientific Position: Prof., Mathematics and its Education, Regional Center of Education and Training, Kenitra, Morocco, Since 2011
Research Interests: Applied Mathematics, Didactics of Mathematics
Scientific Publications: 28 Papers, 2 Books

Name: **Bouazza**
Surname: **El Wahbi**
Birthday: 01.01.1967
Birthplace: Rabat, Morocco
Bachelor: Mathematics, Department of Mathematics, Faculty of Sciences, Mohamed V University, Rabat, Morocco, 1989
Master: Mathematics, Department of Mathematics, Faculty of Sciences, Mohamed V University, Rabat, Morocco, 1993
Doctorate: Department of Mathematics, Faculty of Sciences, Mohamed V University, Rabat, Morocco, 1998
The Last Scientific Position: Prof., Mathematics, Ibn Tofail University, Kenitra, Morocco, Since 2001
Research Interests: Mathematical Analysis and Application, Didactics of Mathematics
Scientific Publications: 35 Papers