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TRANSFORMATION OF DEVELOPABLE SURFACES IN APPLIED TASKS OF CONSTRUCTIONAL PHYSICS

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Abstract- Thanks to the modern development of technical means and ways of managing them, there are numerous opportunities for the use of innovative technologies in architecture, acoustics, solar technology, sun protection and other areas of human activity. Moving parts of the facade, which have been called kinetic facades, are used more and more, the universalization of theaters is achieved due to the transformation of spectator seats and reflective screens that affect acoustic abilities, heliosystems follow the movement of the Sun, increasing their efficiency. This list can be continued. For the design of moving systems, it is important to predict the geometry of changes. We are talking about changing the position and shape of the transformed surfaces. At any moment of the transformation of the surface, the designer must know the geometric and analytical properties of this surface in order to solve the tasks in the given conditions - reflection of sound or light, calculation of bearing capacity, etc. This article considers the transformation of the form of analytical surfaces when applied to problems of constructional physics. In general, non-developable and developable surfaces are subject to transformation for problems of sound or light reflection. In the case of non-developable surfaces, for their transformation it is necessary to divide the surface into smaller sections by parqueting or other methods that can change the metric characteristics of the lines on the surface. The developable surfaces, in special cases cones and cylinders, can be transformed without stretching or breaking, maintaining integrity, but can also be replaced by folds or planar elements. Developable surfaces are most suitable for transformation as ruled surfaces. They have directrixes in the form of surfaces or curves. A single-parametric system of diretrix curves during the transformation gives a double-parametric system of straight lines, that join them, from which the surfaces is allocated for solving the assigned tasks.

Keywords: Developable Surfaces, Surfaces Transformation, Reflective Surface, Congruence, Single Parametric System Set of Lines, Double Parametric System of Lines.

1. INTRODUCTION

Developable surfaces are uniform ruled surfaces of zero Gaussian curvature that can be develop into a plane without compression, breaks, or stretching. In general, these are the general developable surfaces that was researched till the 8th order 4th class T_4^8. In special cases, such surfaces are a cone and a cylinder. In future, let`s call cuspidal edge as C-edge, developable surface as D-surface.

Transformed surfaces can generally be any surfaces, developable or non-developable. Transformation can be two types - a position and a form. The transformation of the position does not change the geometric parameters of the surfaces. Transforming the shape of undevelopable surfaces requires parqueting or other methods, because their transformation changes the metric parameters of the lines on their body. The transformation of the shape of the undevelopable surfaces leaves the lengths of the lines on the body of the developable surface unchanged. There are well-known theorems [1] according to which, when the D-surface is bent into a plane, all its geodesic lines become straight, any point of the C-edge of the tangent surface is the boundary of the intersection point of three infinitely close planes of the D-surface that are tangent to directrices, if the arc of the line is the shortest path between two points of the torso surface, then this line is a geodesic line, two surfaces can be bent into one another when the Gaussian curvature has the same and constant value everywhere on both surfaces. D-surface differs from other ruled surfaces in that two adjacent generatrixes intersect on it, while on other ruled surfaces they cross. If two D-surfaces have common line, then this line belongs to both D-surfaces.

There are studies of geometric and analytical properties of developable surfaces. Known construction methods [1]. It:

- Construction of D-surfaces along two given diretrix curves as a surface that is enveloped by a single-parametric system of planes (roll-up method);
- Frame-parametric method;
- Construction of the D-surface along two lines of curvature of various sets;
- Construction along the given cuspidal edge;
- Kinematic method of winding on a cylinder or cone a straight that lies on plane;

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- Construction according to the law of motion of two points of a rectilinear generatrix;
- Construction of the D-surface as an envelope of a single-parametric system of cones;
- Designing a developable surface based on a spatial closed curve.

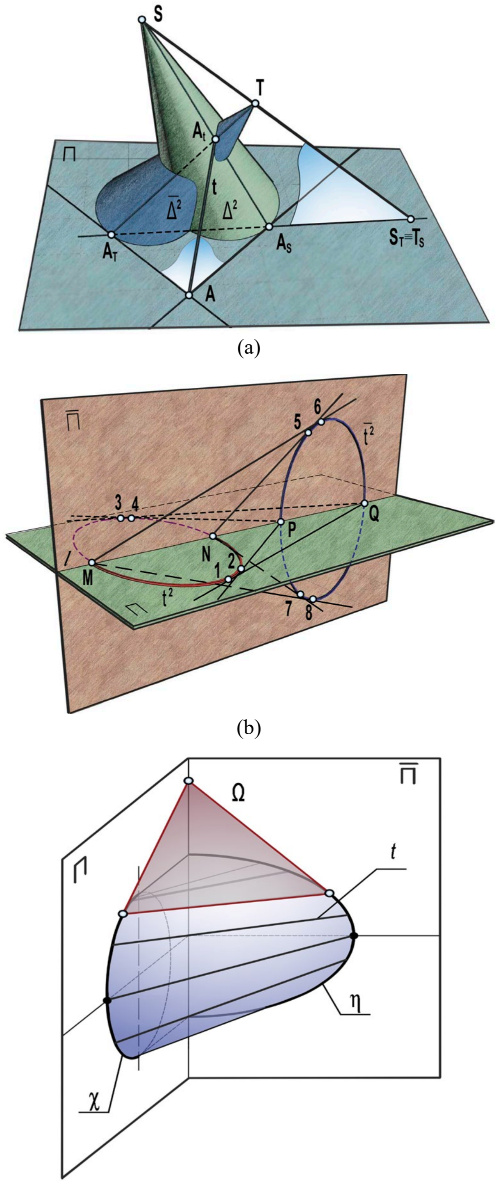
These construction methods can be conditionally reduced to constructions of a cuspidal edge and the running-in method. There are also studies on bending of developable surfaces on a plane [1]. But this is not enough to predict the properties of the developable surfaces from any moment of bending. In other words, the designer at any moment of the transformation of the surface must know the geometric and analytical properties of this developable surface in order to predict the bearing capacity, directionality of reflected solar, light or sound rays, shading properties of the screen.

Developable surfaces refer to the type of surfaces in which the surface normals along rectilinear generatrixes are bundles of parallel straight lines, the tangent planes are envelopes of developable surfaces with contact along rectilinear generatrixes. Accordingly, reflections are flat bundles of straight lines intersecting in an imaginary source at a point source of sound or light. In the case of an improper source, the imaginary source is also improper and given by a vector. By changing the shape of the developable surface reflective screen, you can control the scattering, direction or concentration of the reflected rays. In certain cases, it is needed to receive a reflecting surface when the surface of the reflected rays is known. Such a problem is also solved if we consider the surface transformation as a single-parametric system of developable reflectors.

2. THE ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

Publication [2, 3, 4] determine theoretical and practical basis for constructions the surfaces and it using in construction physics, theory and practice of calculations. Methods of synthetic geometry allows to control visually construction of reflective surfaces and surfaces of reflected rays. The reference book [5] and encyclopedia [1] shows the research of analytical surfaces, their analytical description and visualization. [6, 7] offers a geometric and analytical construction of the reflections in acoustics or insolation. Publication [8] gives an analytical description of the construction of the surface of reflected rays.

Articles [9-13] give methods of calculation of energy efficiency in insolation sphere with methods of applied geometry. Researches [14-17] analyze using solar energy in heating or overheating of buildings. Solar energy spreading in the articles is interpreted in geometric form. Developable surfaces were analyzed in many books, conferences and articles, these researches have a deep history. Sources [18-20] describe classification of developable and non-developable surfaces, methods of their construction, research of the surfaces by methods of synthetic geometry.



(a) (b) (c)

Figure 1. Construction of D-surface, a) construction of C-edge of developable surface; b) estimation of order and class of developable surface; c) construction of developable surface by rolling-up directrix curves

Implementation of researches of developable surfaces were described in sources [21-27]. They give examples of their using in architecture, shading, design, sewing, sculpture, toys design and other spheres.

3. PURPOSE

Finding ways to transform developable surfaces, analyzing methods of constructing developable surfaces for solving applied problems of ray reflection in construction physics.

4. RESULTS

Let us consider in more detail two ways of constructing developable surfaces: the first - by constructing tangent to the intersection line of two 2nd order surfaces, which is the C-edge of the D-surface, or the second, in a dual way, by the method of rolling-up two second-order surfaces or second-order curves.

According to the first method (Figure1a), it is known [2] that the common line of two surfaces of the 2nd order gives the C-edge of the D-surface. Tangents to the C-edge are rectilinear generatrixes of the D-surface. The construction can be considered on the example of the intersection of two cones. Let's draw a cutting plane that passes through the apexes of the cones S and T. The intersection of the generating cones SA_S and SA_S belonging to the cutting plane will give the point at on the lines of intersections of the cones. The intersection of the plane's tangent to the cones along the generatrix lines SA_S and SA_S gives the desired generatrix line of the developable surface, tangent to the C-edge of the D-surface (the line of intersection of the cones).

Analytically, such surfaces are described by Equation (1):

$$f = f(i, w) = a(w) + ib(w) \tag{1}$$

where, $a(w)$ is a radius of C-edge, $b(w)$ is unit tangent vector at every point of the C-edge. The equation of the D-surface or C-edge will take the next form, if we derive the formula of a family of tangent planes $C(x, y, z, w) = 0$:

$$\begin{aligned} a(w) &= xk + yl + zm = \\ &= x(w)k + y(w)l + z(w)m \end{aligned} \tag{2}$$

The principle of duality in projective geometry appears in the fact that each position of points, planes, and lines in space corresponds to another position by replacing points with planes, planes with points, while the same line corresponds to a straight line. Accordingly, a series of points lying on one straight line is dual to a bundle of planes, a field of points lying on one straight line corresponds to a dual bundle of planes, and the same bundle of straight lines corresponds to a bundle of straight lines.

The construction of the D-surface on a basis of C-edge is dual to the construction by the rolling-up method. A bundle of planes that are in one-to-one correspondence and tangent to a pair of cones is the dual to locus of the points of curves of the 2nd order, which are also in one-to-one correspondence, i.e., the bundle of planes tangent to both curves circumflex developable surface.

The method of rolling-up with a single-parametric family of planes involves the establishment of a one-to-one correspondence between the points of the rolling-up objects, rectilinear elements of the developable surface are created by combining these points. Construction of the developable surface is possible using the displacement of a line along two diretrix curves:

$$\begin{aligned} &r_1(\chi) = f_1(\chi)k + F_1(\chi)l + \chi m \\ &r_2(\eta)k + F_2(\eta)l + \eta m \end{aligned} \tag{3}$$

Given that $z = \chi$ denotes a point of the 1st directrix curve, $z = \eta$ denotes a point on the 2nd directrix curve. In order for both curves be touched by the rolling-up plane simultaneously, a condition must be:

$$\begin{vmatrix} f_1'(\chi) & F_1(\chi) & 1 \\ f_2'(\eta) & F_2(\eta) & 1 \\ f_1'(\chi) - f_2'(\eta) & F_1(\chi) - F_2(\eta) & \chi - \eta \end{vmatrix} = 0 \tag{4}$$

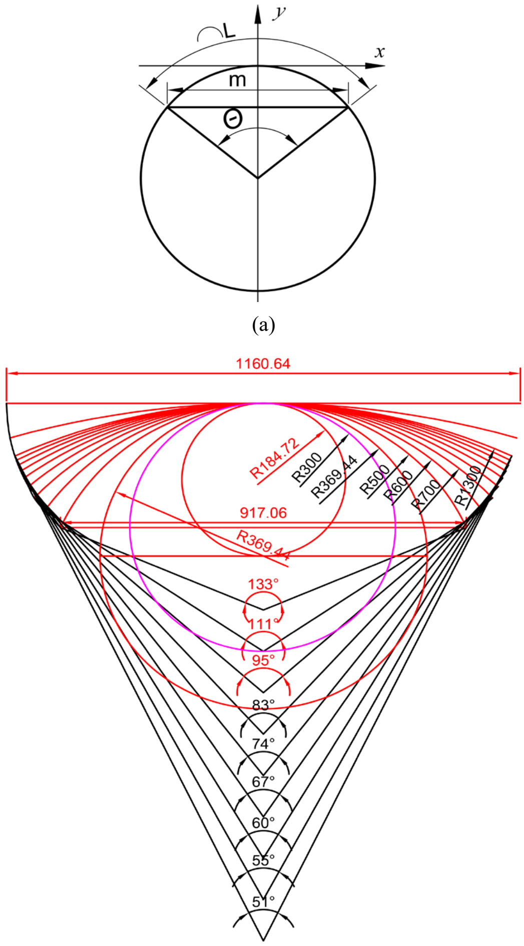


Figure 2. Drawings for equation of trajectory of arc points drawing for equation of movement of arc points single parametric system of arcs of equal length

According to the classification, the developable surfaces have a class and an order. Developable surfaces of 4th class can be obtained as a surface that can be created be enveloping by the bundle of planes of 4th order. These planes are tangent simultaneously to two non-ruled or ruled undevelopable surfaces of the 2nd order Ω^2 and Ω^2, or a surface a curve of the 2nd order both - Ω^2 and t^2, or two curves of the 2nd order t^2 and t^2. That is, through an arbitrary point A in space, four planes simultaneously tangent to two figures can be drawn. Thus, with the general position of the rolling-up figures (Figure 1, b), the formed developable surface has the 4th class. Generatrixes of developable surfaces are lines that connect tangents points of plane for two figures. The order of the developable surface is determined by the total order of the lines in the plane of the rolling-up curve. With the general position of two conics (ellipses in the example), in the plane of the rolled-up curve there are four tangents to the ellipse and the double ellipse itself, which is the 8th order of the developable surface. When changing the relative location of the rolling-up curves, the order and class of the surface will change. The cone and the cylinder are also developable surfaces of the 2nd order. For some types of transformation, it is convenient to use them as basic transformation surfaces.

Consider the developable surface created by the rolling-up method. In the general case, curvilinear rolling-up generatrixes can be located in parallel or non-parallel planes. To construct the developable surface graphically, it is necessary to draw tangents to the curves from the point that bolongs to the intersection line of the planes Π and $\overline{\Pi}$. These planes are planes of our diratrix lines. The lines t will form the developable surface. Construction according to this rule is shown in Figure 1c.

Changing the position of the rolling-up curves or their shape will lead to a change in the shape of the developable surface. The first case is a change in the position of the rolling-up curve (or two curves). There are four options for changing the position: parallel movement in the plane of the direcnrix, rotation in the plane of the diretrix, movement parallel to the plane of the directrix, rotation from the plane of the directrix. In the last option, the angle between the planes of the curve's changes. There are studies that show how the class and order of the developable surfaces change when the position of the rolling-up curves changes [2].

For the applied use of developable surfaces in the case of changing the position of the rolling-up curves, it is necessary to investigate how the position of the rectilinear generatrixes changes during the transformation and what traces on the body of the D-surface the points of the rolling-up curve describe.

The second case is a change in the shape of the rolling-up curve. If in the first case we have constant rolling-up curves, for example, ellipses, which change the parameters of the developable surface due to the mutual location of the curves, then in the second case, when changing the shape of the rolling-up curves, it is necessary to provide analytically and geometrically which curves are the directrix lines of the rolling-up at each moment of transformation. Know that an ellipse is transformed into an ellipse with known parameters, transformed into another curve.

As an example, you can consider the bending of a circle. In applied problems, to implement such a transformation, it is necessary to know the trajectories along which each point of the circle moves. But not only that is important. On this trajectory, the points of the rolling-up curve are fixed to build the desired shape of the curve, with known characteristics that, when built by the rolling-up method, gives a developable surface with the desired properties. Analytical and geometric constructions of the trajectory make it possible to predict the shape of the rolling-up curve at any time. The parametric equation of the trajectory of the points of the circle arc will take the form:

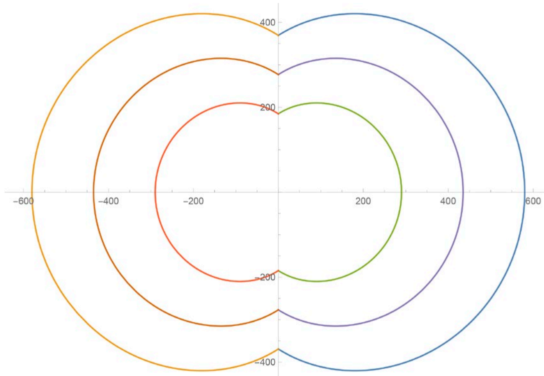
$$\begin{aligned} x &= \frac{L}{\theta}\sin\frac{\theta}{2} \\ y &= r - \frac{L}{\theta}\cos\frac{\theta}{2} = \frac{L}{\theta} - \frac{L}{\theta}\cos\frac{\theta}{2} \end{aligned} \tag{5}$$

where, L is arc lenght, and θ is angle of the arc (Figure 2a).

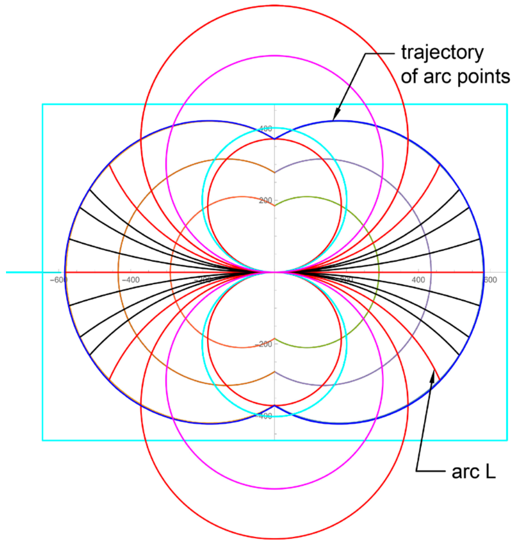
Geometric constructions (Figure 2b) appear in the variation of the radius of the arc, which does not change its length during bending. Using the well-known formulas of the arc of a circle, for the graphical construction of the movement of the points of the circle during bending, Table 1 is calculated.

Table 1. Characteristics of arcs of circle during transformation

Radius r	184.72	200	300	400	500
Central angle α, degree	360.0	332.5	221.7	166.3	133.0
Central angle θ, radians	6.3	5.8	3.9	2.9	2.3
Length of an arc L	1160.6	1160.6	1160.6	1160.6	1160.6
Length of chord m	0.0	95.1	560.8	794.2	917.1



(a)



(b)

Figure 3. Examples of visualization of trajectory of arc points during transformation, a) visualization in Mathematica, b) overlay of single parametric system of arcs to Mathematica visualization

To check the results of the parametric equation, the trajectories of the points of the arc are constructed in the Mathematica program (Figure 3a). According to the algorithm of geometric constructions, the elements of a single-parametric system of arcs are drawn, the results are superimposed on the construction of the Mathematica program (Figure 3b), which shows the coincidence. Thus, having transformed circles as rolling-up curves, we will obtain a developable surface with known predetermined geometric characteristics, which allows us to investigate the properties of the resulting surface as a reflective screen (Figures 4a, 4b).

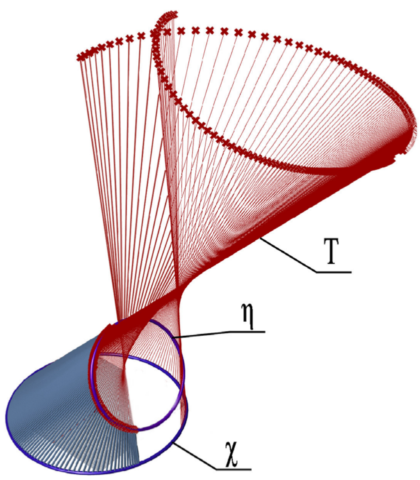
Construction of reflections from the developable surface occurs in two ways: with the help of an imaginary source or due to the symmetry of the reflected and incident rays relative to the surface normal. With a point source, the geometric location of the points of the imaginary sources is created thanks to the single-parametric family of the rolling-up planes of D-surface. Reflections from rectilinear generators of the developable surface will represent a single-parameteric system of flat beams centered at imaginary sources. With a non-proper source, the surface of reflections is a flat bundle of parallel lines. In the analytical description, is given by a vector.

The second construction method involves constructing the reflected ray symmetrically to the normal at the point of reflection. According to the classification of reflecting surfaces by the principle of surfaces of normals to the generatrixes of surfaces [6], the developable surface belongs to the first group, where the surface of normals to the rectilinear generatrixes is a flat bundle of parallel lines. Reflections constructed in this way are a flat beam of rays with the point of origin in an imaginary source for a point source or a flat beam of parallel lines for a non-proper source. Analytically, a beam of reflected rays from a rectilinear generatrix of developable surface in a parametric view has the form:

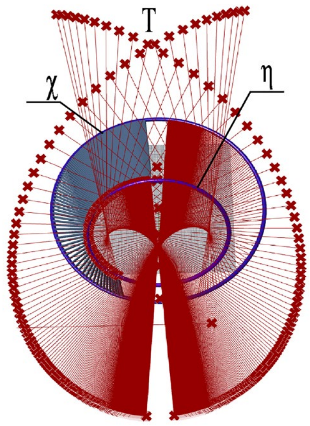
$$\begin{aligned} x &= x_{S'} + t\left(x_a - x_{S'} - k\left(x_\chi - x_\eta\right)\right) \\ y &= y_{S'} + t\left(y_a - y_{S'} - k\left(y_\chi - y_\eta\right)\right) \\ z &= z_{S'} + t\left(z_a - z_{S'} - k\left(z_\chi - z_\eta\right)\right) \end{aligned} \tag{6}$$

where $x_\beta, y_\beta, z_\beta$ are coordinates of an arbitrary point of the rolling-up curve β, x_η, y_η, z_η are coordinates of the corresponding point of the rolling-up curve γ, which are connected by a unique correspondence, $x_{S'}, y_{S'}, z_{S'}$ are coordinates of an imaginary source, the parameters of which depend on the three coefficients of the plane A, B, C, tangent to reflecting surface at the point A (x_a, y_a, z_a).

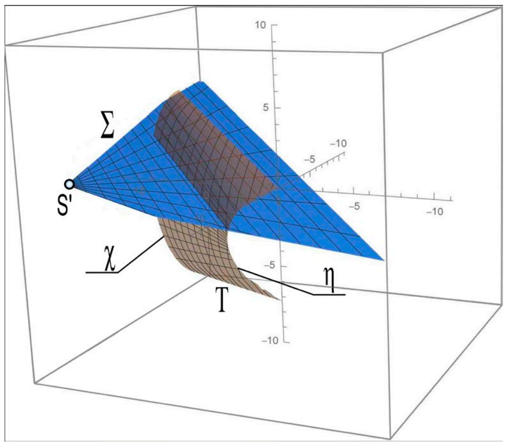
$$A = \begin{vmatrix} y'_\chi & z'_\chi \\ y'_\eta & z'_\eta \end{vmatrix},\ B = \begin{vmatrix} z'_\chi & x'_\chi \\ z'_\eta & x'_\eta \end{vmatrix},\ C = \begin{vmatrix} x'_\chi & y'_\chi \\ x'_\eta & y'_\eta \end{vmatrix} \tag{7}$$



(a)



(b)



(c)

Figure 4. Developable surface, a) constructed in Grasshopper, perspective view; b) constructed in Grasshopper, plan view; c) developable surface and surface of reflected rays with center in imaginary source constructed in Mathematica.
T: developable surface, ∑: surface of reflected rays
S`: imaginary source, β and γ: rolling-up curves

Having the equation of the surface of the reflected rays, it is possible to predict the nature of the reflections, scattering and concentration or to visualize the reflections (Figure 4c). The selection of individual reflected rays from a single-parametric ssystem makes it possible to establish correspondence between an arbitrary point in space where the reflected ray arrives and the point of reflection on the surface. This makes it possible to outline the contours of the reflective area, predict the physical properties of the reflection energy transfer to a certain point, and more.

In the case of reflections, the transformation of the developable surface allows dynamic redirection of the reflection, which can be used in acoustics calculations, as well as the reflection of light and sunlight. In these cases, it is possible to operate with the reflected energy for its concentration in cases of work with solar collectors, scattering for the purposes of combating overheating, protection from the sun when designing shading, or for diffuse distribution during acoustic calculations. Further research aims to expand the possibilities of applying the transformation of developable surfaces for use in the field of architectural design, acoustics, sun protection, lighting, solar systems and other tasks.

5. CONCLUSIONS

The article considers developable surfaces as transformed, i.e., such that, changing their shape, retain the given geometric characteristics. From a geometric point of view, a transformation is a single-parametric system of developable surfaces. However, in a broader sense, the transformation can be carried out into another type of developable surfaces with different directrix curves. When obtaining developable surfaces by the rolling-up method, a single-parametric system of surfaces is simultaneously tangent to two rolling-up surfaces, a curve and a surface or two curves. The transformation of the developable surface occurs due to the change in the shape of the directrix curves. It is extremely important to bend curves in such a way that at any moment in time you know the parameters of this curve, its equation, and position in space. Then the parameters of the developable surface, which are shaped by the directrix curves, are known.

The proposed method of bending a circle with constant length. According to the proposed method, it is possible to develop mechanical models of circle bending with predicted parameters. Analytical and geometric methods of constructing developable surfaces are shown and tested in computer programs.

The equations of the surfaces of reflected rays from developable surfaces are derived, the possibilities of using the transformation of developable surfaces and reflections from them in architectural design, in solving the problems of acoustics, heliotechnics, sun protection are outlined.

The transformation of the developable surfaces allows you to solve dynamically the reflection problems, redirecting the energy in accordance with the existing needs and the change of the environment.

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Scientific Memberships: Ukrainian Association of Applied Geometry

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The Last Scientific Position: Prof., Department of Architectural Constructions, Architecture Faculty, Kiev National University of Construction and Architecture, Kiev, Ukraine, Since 2011
Research Interests: Building Physics, Energy Efficient Buildings, Solar Energy, Creation of Building Codes for Energy Efficiency
Scientific Publications: 202 Papers, 10 Books, 8 Patents, 30 National Building Standards, 62 Projects
Scientific Memberships: National Academy of Higher Education of Sciences of Ukraine, Ukrainian Academy of Architecture

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The Last Scientific Position: Assistant, Department of Architectural Constructions, Architecture Faculty, Kiev National University of Construction and Architecture, Kiev, Ukraine, Since 2006
Research Interests: Architectural Constructions, Energy Efficient Buildings, Solar Energy, Churches Architecture
Scientific Publications: 8 Papers, 4 Manuals, 5 Theses, 105 Projects
Scientific Memberships: Ukrainian Association of Applied Geometry

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Master: Architecture of Buildings and Constructions, Architecture Faculty, Kiev National University of Construction and Architecture, Kiev, Ukraine, 2006
Ph.D.: Applied Geometry, Engineering Graphics, Department of Architectural Constructions, Architecture Faculty, Kiev National University of Construction and Architecture, Kiev, Ukraine, 2021
The Last Scientific Position: Assoc. Prof., Department of Architectural Constructions, Architecture Faculty, Kiev National University of Construction and Architecture, Kiev, Ukraine, Since 2021
Research Interests: Architectural Constructions, Energy Efficient Buildings, Solar Energy, Applied Geometry
Scientific Publications: 49 Papers, 1 Book, 11 Theses, 5 Manuals, 1 National Building Standard, 30 Projects
Scientific Memberships: Ukrainian Association of Applied Geometry